

Example 28. (Example 3.8 on textbook page 100) Six boxes of components are ready to be shipped by a certain supplier. The number of defective components in each box is as follows:

Box	1	2	3	4	5	6
Number of defectives	0	2	0	1	2	0

One of these boxes is to be randomly selected for shipment to a particular customer. Let X be the number of defectives in the selected box.

x	0	1	2
$P(x)$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$

check $\sum = 1$ ✓
 $0 \leq \leq 1$ ✓

Lec 10

3.2.2 The Cumulative Distribution Function

Definition 10. The cumulative distribution function (cdf) $F(x)$ of a random variable X is

$$F(x) = P(X \leq x)$$

properties:

- 1) $0 \leq F(x) \leq 1$
- 2) non decreasing:
 increasing x does not decrease $F(x)$

Example 29. Experiment: rolling an unfair die

Define the random variable X as the number on the upper face. Then the pmf of X is given in the table

x	1	2	3	4	5	6
$p(x)$	0.2	0.3	0.1	0.1	0.1	0.2

Then some of the probabilities we are interested in are

$$F(1) = P(X \leq 1) = P(X=1) = p(1) = 0.2$$

$$F(2) = P(X \leq 2) = P(X=1 \text{ or } 2) = p(1) + p(2) = 0.2 + 0.3 = 0.5$$

$$F(3) = 0.2 + 0.3 + 0.1 = 0.6$$

$$F(4) = 0.7 \quad F(5) = 0.8 \quad F(6) = 1$$

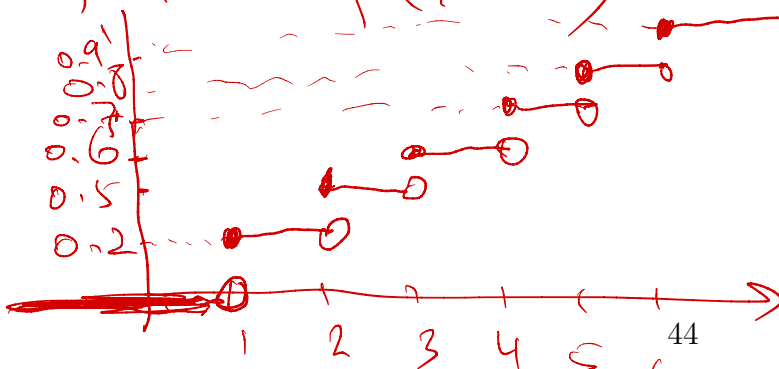
$$F(-0.5) = 0 \quad F(0) = 0 \quad F(0.5) = 0$$

$$F(1.5) = P(X \leq 1.5) = P(X=1) = 0.2$$

$$F(x) = P(X=1) = 0.2 \quad 1 \leq x < 2$$

$$F(x) = P(X \leq x) = 0$$

$$x < 1$$



notice:

right-continuous.

jumps at possible values of X
by the prob. of the value

constant in between possible val.

$$F(x) = \begin{cases} 0 & \text{if } x < 1 \\ 0.2 & \text{if } 1 \leq x < 2 \\ 0.5 & \text{if } 2 \leq x < 3 \\ 0.6 & \text{if } 3 \leq x < 4 \\ 0.7 & \text{if } 4 \leq x < 5 \\ 0.8 & \text{if } 5 \leq x < 6 \\ 1 & \text{if } x \geq 6 \end{cases}$$

$$P(1 \leq X \leq 3) = P(X=1, 2, \text{ or } 3) \quad \checkmark$$

$$= 0.2 + 0.3 + 0.1 = 0.6$$

BCA $\begin{matrix} X \leq 3 \\ \text{not } X < 1 \end{matrix}$

$$P(A \setminus B) = P(A) - P(B)$$



$$\rightarrow P(X \leq 3) - P(X < 1) = 0.6 - (0.2 - 0.2) = 0.6$$

\uparrow 0.6 \uparrow $X \leq 1$ not = 1 \swarrow 0.6
 $P(X \leq 1) - P(X=1)$

$$P(1.5 < X \leq 3.5) = F(3.5) - F(1.5) \\ X \leq 3.5 \text{ not } \leq 1.5 = 0.6 - 0.2 = 0.4$$

$$P(2 \leq X < 4) = (F(4) - \text{jump}) - (F(2) - \text{jump}) \\ X < 4 \text{ not } X < 2 = 0.6 - 0.2$$

$$P(2 \leq X \leq 5) = 0.8 - 0.2 \\ X \leq 5 \text{ not } X < 2$$

$$P(2 < X < 5) = 0.7 - 0.5 \\ X < 5 \text{ not } \leq 2$$

Example 30. Continue with our example of tossing a fair coin three times in Example 26. Find the CDF of X .

x	0	1	2	3
$p(x)$	0.125	0.375	0.375	0.125

Solution.

Do it

Question: Given a CDF, how do we convert to pmf?

In Example 30,

Look at jumps

locations give possible values

sizes give $p(x)$

Example 31. (Example 3.13 from textbook page 104) A store carries flash drives with either 1 GB, 2 GB, 4 GB, 8 GB, or 16 GB of memory. The accompanying table gives the distribution of Y = the amount of memory in a purchased drive:

y	1	2	4	8	16
$p(y)$	0.05	0.1	0.35	0.4	0.1

Draw the CDF
and write its formula