Station 1 had 2 pumps

2 had 3 pumps

If this experiment is performed and results $\omega = (2, 3)$, then

$$X = 2+3=5$$

$$Y = 2-3=-1$$

$$U = 3$$

Example 24. When a student calls a university help desk for technical support, he/she will either immediately be able to speak to someone (S, for success) or will be placed on hold (F, for failure). With $S = \{S, F\}$, define an random variable X by

$$X(S) = 1 \text{ and } X(F) = 0$$

The random variable X indicates ________________________________

Example 25. Suppose a location in the United States is selected. Define the random variable Y by

$$Y = \text{ the height above sea level at the selected location}$$

Then the largest possible value of Y is 14,494 (Mt. Whitney), and the smallest possible value is 2282 (Death Valley). The set of all possible values of Y is the set of all numbers in the interval between 2282 and 14,494, that is, $[2282, 14494]$

and there are uncountably (infinitely) many values in this interval.

Lec 9

Two Types of Random Variables

- Discrete. If the possible outcomes of a random variable can be listed out using a finite (or countably infinite) set of single numbers (Example 22, 23, 24), then the random variable is discrete.
- continuous. If the possible outcomes of a random variable can only be described using an interval or union of intervals of real numbers (Example 25), then the random variable is continuous.

3.2 Probability Distributions for Discrete Random Variables

3.2.1 The Probability Mass Function

The **probability distribution** of X says how the total probability of 1 is distributed among the various possible X values. The probability distribution of X lists all possible values of X and their corresponding probabilities.

Definition 9. For discrete random variables, the probability list of X is called probability mass function (pmf), which is defined for every number x by $p(x) = P(X = x) = P(\text{all } \omega \in S : X(\omega) = x)$.

The pmf returns the probability that the random variable X is equal to the value x.

To be a valid pmf, we need:

(1) $0 \le p(x) \le 1$ for all x

(2) $\sum_x p(x) = 1$

Example 26. Suppose we toss a fair coin three times, and define the random variable X to be the number of heads that appear. Find the pmf of X.

Solution. The sample space is

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

So the possible values for the random variable X are in the set $\{0, 1, 2, 3\}$. The pmf tells us all possible values of X and their corresponding probabilities, i.e. $p(x) = P(X = x)$. Since we have a fair coin, so the ________________________________ is assumed here. Therefore

x	0	1	2	3
$p(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

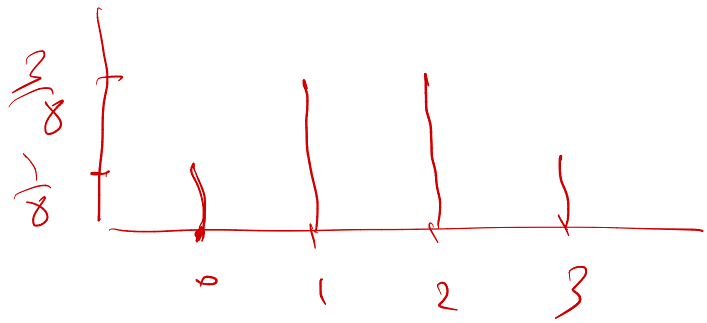
NOTE: This is NOT a proper format of writing a pmf. Write it in a proper way, X should define on ________________________. So The pmf of X is given by

$$p(x) = \begin{cases} \frac{1}{8} & \text{if } x = 0 \text{ or } 3 \\ \frac{3}{8} & \text{if } x = 1 \text{ or } 2 \\ 0 & \text{o/w} \end{cases}$$

Question: Is this a valid pmf?

Check: values b/w 0 & 1 and $\sum = 1$ ✓

The graph pf the pmf of X could be



A event $A = \bigcup_{x \in A} \{x\}$ ← countably many → $\boxed{P(A) = \sum_{x \in A} p(x)}$

Example 27. (Exercise #13 on page 107 of the textbook) A mail-order computer business has six telephone lines. Let X denote the number of lines in use at a specified time. Suppose the pmf of X is as given in the accompanying table.

x	0	1	2	3	4	5	6
$p(x)$	.10	.15	.20	.25	.20	.06	.04

Check: $0 \le \cdot \le 1$ ✓ $\sum = 1$ ✓

Calculate the probability of each of the following events.

a. {at most three lines are in use} $= \{0, 1, 2, 3\} : 0.1 + 0.15 + 0.2 + 0.25 = 0.70$

b. {fewer than three lines are in use} $= \{0, 1, 2\} : 0.45$

c. {at least three lines are in use} $= \{3, 4, 5, 6\} = \{0, 1, 2\}^c : 1 - 0.45$

d. {between two and five lines, inclusive, are in use } $\{2, 3, 4, 5\} : \ldots$

e. {between two and four lines, inclusive, are not in use} $= \{2, 3, 4\} : \ldots$

f. {at least four lines are not in use} $= \{0, 1, 2\} : 0.45$

Solution. Before we calculate the probabilities, let's check whether it is a valid pmf.

Example 28. (Example 3.8 on textbook page 100) Six boxes of components are ready to be shipped by a certain supplier. The number of defective components in each box is as follows:

Box	1	2	3	4	5	6
Number of defectives	0	2	0	1	2	0

One of these boxes is to be randomly selected for shipment to a particular customer. Let X be the number of defectives in the selected box.

x	0	1	2
$p(x)$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$

check $\Sigma = 1$ ✓

$0 \leq \ \leq 1$ ✓

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3.2.2 The Cumulative Distribution Function

Definition 10. The **cumulative distribution function (cdf)**