2.4 Conditional Probability

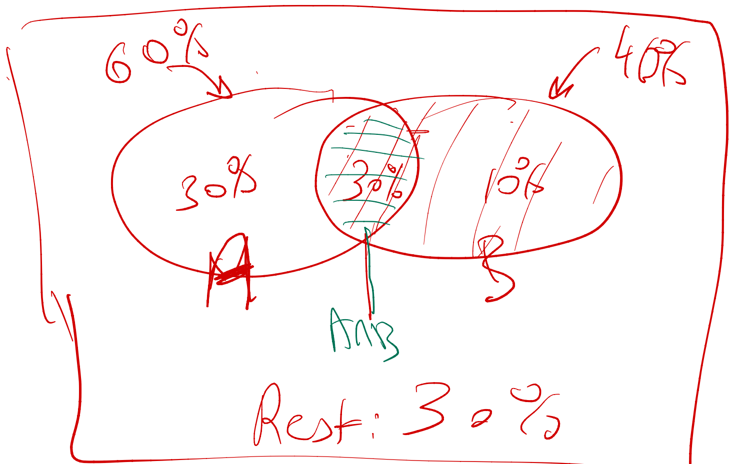
In this section, we examine how the information "an event B occurred" affects the prob. of A. We will use the notation $P(A|B)$ to represent the conditional prob. of A given B

Definition 5. For any two events A and B with $P(B) > 0$, the **conditional probability of** A **given that** B **has occurred** is defined by

$$\frac{\frac{\# A \cap B}{n}}{\frac{\# B}{n}} \longrightarrow \boxed{\frac{P(A \cap B)}{P(B)} = P(A|B)}$$

Example 13. (Example 2.25 from textbook page 76) Suppose that of all individuals buying a certain digital camera, 60% include an optional memory card in their purchase, 40% include an extra battery, and 30% include both a card and battery.

Consider randomly selecting a buyer and let A = memory card purchased and B = battery purchased.



$$P(A) = 0.6$$

$$P(B) = 0.4$$

$$P(A \cap B) = 0.3$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.4} = 0.75 = 75\%$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.3}{0.6} = \frac{1}{2}$$

$$P(A|B) \neq P(B|A) \quad \text{(in general)}$$

Move it earlier in 2.3 " More probability properties part"

(5)

DeMorgan's Laws:

1. $(A \cap B)^c = A^c \cup B^c$
2. $(A \cup B)^c = A^c \cap B^c$

e.g. $(A \cap (B^c \cup C) \cap D)^c$

$= A^c \cup (B \cap C^c) \cup D^c$

Thus,

$$P(A^c \cap B^c) = 1 - P(A \cup B)$$

$(A \cup B)^c$

(6) By distributive law we have

$$A = (A \cap B) \cup (A \cap B^c)$$

Since $(A \cap B)$ and $(A \cap B^c)$ are disjoint, by Probability Axiom 3, we have

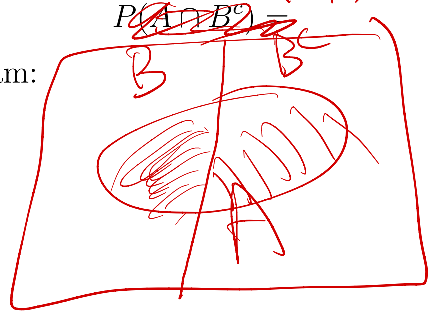
$$P(A) = P(A \cap B) + P(A \cap B^c)$$

$$= P(A|B)P(B) + P(A|B^c)P(B^c)$$

This gives

$$\cancel{P(A \cap B^c) =}$$

Venn diagram:



$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$\rightarrow P(A \cap B) = P(B)P(A|B)$

similarly:

$P(A \cap B^c) = P(B^c)P(A|B^c)$

End

Law of total probability

Eg. Drawing 2 cards (one then the next) from a deck of 52.

$P(\underline{\text{second is King}})$ B = first is K

A

$P(2^{nd}\ K) = P(1^{st}\ K \cap 2^{nd}\ K) + P(1^{st}\ \text{not}\ K \cap 2^{nd}\ K)$

$$P(1^{st}K \cap 2^{nd}K) = P(1^{st}K)\,P(2^{nd}K \mid 1^{st}K)$$

$$= \frac{4}{52} \times \frac{3}{51}$$

$$P(1^{st}\text{ not }K \cap 2^{nd}K) = P(1^{st}\text{ not }K)\,P(2^{nd}K \mid 1^{st}\text{ not }K)$$

$$= \frac{48}{52}\,\frac{4}{51}$$

$$P(2^{nd}K) = \frac{4}{52}\cdot\frac{3}{51} + \frac{48}{52}\cdot\frac{4}{51}$$

Also:

If $B \subset A$ (B is inside A)

Then:

$$P(A\setminus B) = P(A) - P(B)$$

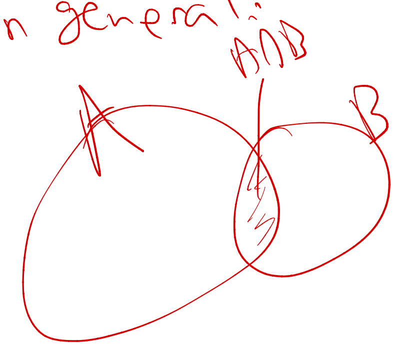
because: $A\setminus B$ and B are disjoint

and $(A\setminus B)\cup B = (A\cap B^c)\cup B = \underbrace{(A\cup B)}_{A}\cap\underbrace{(B^c\cup B)}_{S}$

$= A\cap S = A$

so $P(A) = P((A\setminus B)\cup B) = P(A\setminus B) + P(B)$

In general: $P(A\setminus B) = P(A) - P(A\cap B)$



$\left(\begin{array}{c} A\cup B = B \\ \text{if } B\subset A \end{array}\right)$