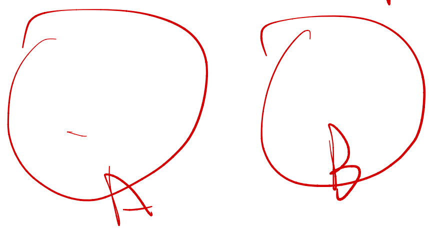
If events A and B are mutually exclusive, then $A \cap B = \emptyset$

Venn diagram:



$S = \{0, 1, 2, 3, 4, 5, 6\}$

Example 8. For the experiment in which the number of pumps in use at a single six-pump gas station is observed, let $A = \{0,\ 1,\ 2,\ 3,\ 4\}$, $B = \{3,\ 4,\ 5,\ 6\}$, and $C = \{1,\ 3,\ 5\}$. Then

$$A^c = \{5, 6\}$$

$$A \cup B = \{0, 1, 2, 3, 4, 5, 6\}$$

$$A \cup C = \{0, 1, 2, 3, 4, 5\}$$

$$A \cap B = \{3, 4\}$$

$$A \cap C = \{1, 3\}$$

$$(A \cap C)^c = \{0, 2, 4, 5, 6\}$$

Lec 5

2.2 Axioms, interpretations, and Properties of Probability

Interpreting Probability

Consider an experiment that can be repeatedly performed in an indep. & identical manner and let A be an event consisting of a set of outcomes of the experiment. For examples, the coin-tossing experiment previously discussed. If the experiment is performed n times, let #A denote the number times ~~of~~ at which A occurs. Then the ratio $\frac{\#A}{n}$ is called the relative frequency of the event A in the sequence of n replications.

Given an experiment, the objective of probability is to assign to each A a number, called the probability of the event A, which will give a precise measure of the chance that A will occur.

Relative frequency will stabilize as the number of replications n increases. That is, as n gets arbitrarily large, $\frac{\#A}{n} \xrightarrow[n\to\infty]{}$ probability of A.

The objective interpretation of probability identifies this limiting relative frequency with $P(A)$, i.e. as $n \to \infty$

$$P(A) = \lim_{n\to\infty} \frac{\#A}{n}$$

NOTE: $0 \le P(A) \le 1$. Why?

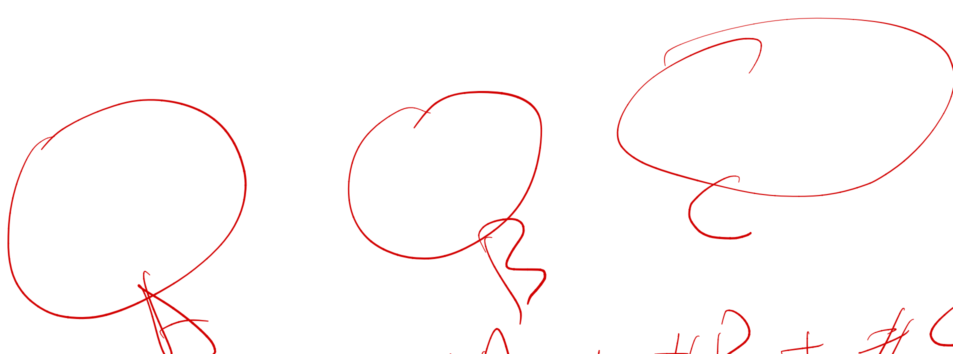
The assignment of probabilities should satisfy the following axioms of probability.

Probability Axioms

1. $0 \le P(A) \le 1$
2. $P(S) = 1$
3. If A_1, A_2, $\ldots$ is an infinite collection of disjoint events, then

pairwise

$$P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$$



$$\#(A \cup B \cup C) = \#A + \#B + \#C$$

so: $P(A \cup B \cup C) = P(A) + P(B) + P(C)$

A, A^c disjoint

$A \cup A^c = S$

$1 = P(S) = P(A) + P(A^c)$

More Probability Properties

(1) $P(\emptyset) = 0$

(2) $P(A) \leq 1$

(3) $P(A^c) = 1 - P(A)$

$P(A^c) = \frac{1}{2}$

Example 9. Suppose we flip a fair coin until a head appears for the first time. What is the probability that more than one flip of the coin is required?

Solution. The sample space S is

$$S = \{H, TH, TTH, TTTH, \ldots\}$$

So the event A=more than one flip of the coin is required contains the outcomes

$$A = \{TH, TTH, \ldots\} \qquad A^c = \{H\}$$

Since each of the outcomes in A ~~are~~ can not occur simultaneously, ______________

$A = \{TH\} \cup \{TTH\} \cup \{TTTH\} \cup \ldots$

Therefore,

$$P(A) = P(TH) + P(TTH) + \ldots$$

$P(TH) = \frac{1}{4} = \frac{1}{2^2}$ $\quad P(TTH) = \frac{1}{8} = \frac{1}{2^3}$ $\quad P(TTTH) = \frac{1}{2^4}$

However, if we use property (3), we can solve the problem very quick by noticing that

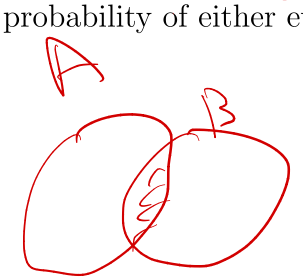
$$P(A) = \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \ldots = \frac{\frac{1}{2^2}}{1 - \frac{1}{2}} = \frac{1}{4} \times 2 = \frac{1}{2}$$

$A^c = \{H\}$

$P(A^c) = \frac{1}{2}$

$P(A) = 1 - P(A^c) = \frac{1}{2}$

(4) For any two events A and B, probability of either event A **or** event B occurring is



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- Special case:

$A \cap B = \emptyset$

$\Downarrow$

$P(A \cap B) = 0$

$\Rightarrow P(A \cup B) = P(A) + P(B)$

Example 10. (Example 2.14 from textbook page 62) In a certain residential suburb, 60% of all households get Internet service from the local cable company, 80% get television service from that company, and 50% get both services from that company. If a household is randomly selected,

Question:
1. what is the probability that it gets at least one of these two services from the company?

2. what is the probability that it gets exactly one of these services from the company?

Solution.

1) $P(I \cup T) = P(I) + P(T) - P(I \cap T)$

$= 0.6 + 0.8 - 0.5$

$= 0.9$

2) $P((I \setminus T) \cup (T \setminus I)) = 0.4$

$+ P(I \setminus T) + P(T \setminus I) \leftarrow 0.8 - 0.5 = 0.3$

$0.6 - \cancel{0.8}\ 0.5 = 0.1$

$I \setminus T = I \setminus (I \cap T)$