Lec 4

2 Probability

The term **probability** refers to the study of uncertainty

With a small number of observations, outcomes of random phenomena may look quite different from what you expect. As we make more observations, the proportion of times that a particular outcome occurs gets closer and closer to a certain number we would expect.

With any random phenomena, the probability of a particular outcome is the proportion of times would occur in a long run of observations.

NOTE: A random phenomenon has the characteristic that ~~it~~ is predictable in the long run.

2.1 Sample Spaces and Events

An **experiment** is any activity of process whose outcomes are subject to uncertainty.

Experiments that may be of interest include:

- tossing a coin once or several times,
- selecting a card or cards from a deck,
- weighing a loaf of bread.

Definition 2. The **sample space** of an experiment, denoted by S, is the set of all possible outcomes of the experiment

Example 6.

- Experiment: Observing the tosses of two fair coins.

  $S = \{HH, HT, TH, TT\}$

- Experiment: Flip a fair coin until a tail appears for the first time

  $S = \{T, HT, HHT, \ldots\}$

- Experiment: Flip a fair coin until the first tail and record the number of heads that have occurred.

  $S = \{0, 1, 2, 3, \ldots\}$

- Experiment: Observe the highest temperature for today:

  $S =$ all real numbers ($\mathbb{R}$)

- Experiment: Randomly select an American household and record the number of TV sets.

  $S = \{0, 1, 2, 3, 4, \ldots\}$

In our study of probability, we will be interested not only in the individual outcomes of S

but also in various collections of outcomes.

> **Definition 3.** An **event** is any collection of outcomes from sample space.
>
> An event is Simple if it consists of exactly one outcome and compound
>
> if it consists of more than one outcome.

When an experiment is performed, a particular event A is said to occur if the resulting outcome was in the event.

Example 7. Experiment: Tossing a coin 3 times. The sample space is

$S = \{HHH, HHT, HTH, HTT, \ldots\}$ (8 of them)

Suppose our event

$$A = \text{ First toss gives head.}$$

Then A occurs only if the resulting experimental outcome is contained in the set

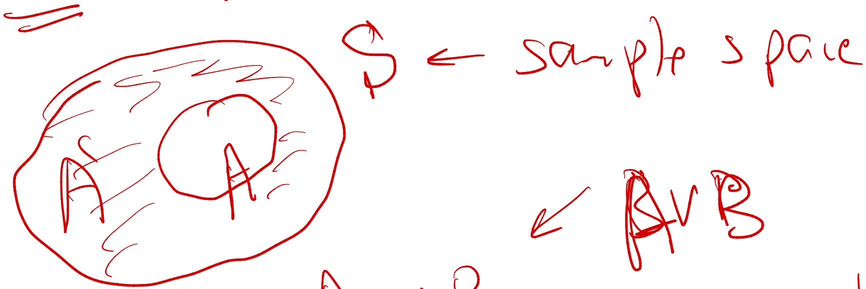
$A = \{HHH, HHT, HTH, HTT\}$

Some relations from Set theory An event is just a set, so relationships and results from elementary set theory can be used to study events.

Definition 4.

- The **complement** of an event A, denoted by A^c, is the set of outcomes not in A.

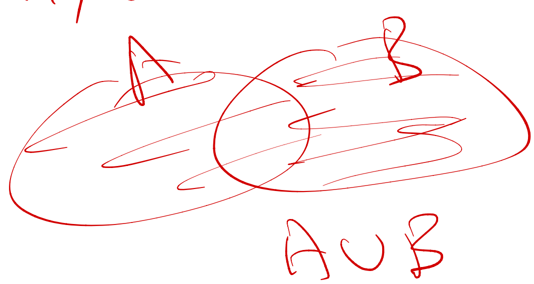
  Venn diagram:



- The **union** of two events A and B, denoted by $A \cup B$, is the event consisting of outcomes in A or B or both

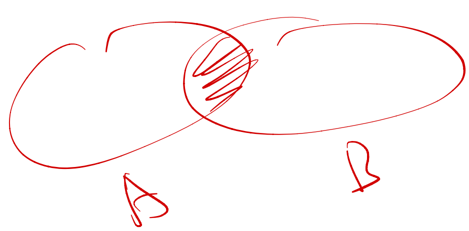
  So the union includes outcomes for which both A and B occur as well as outcomes for which exactly one occurs, that is, all outcomes in at least one of A and B.

  Venn diagram:



- The **intersection** of two events A and B, denoted by $A \cap B$, is the event consisting of outcomes in both A and B

  Venn diagram:

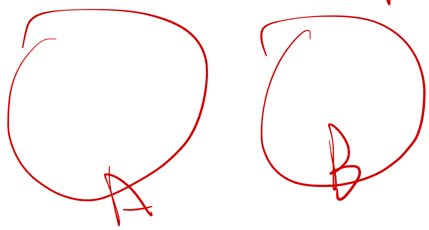


empty set $\emptyset$

- Two events are **mutually exclusive or disjoint**, if they have no outcomes in common, i.e. both events can not happen at the same time

If events A and B are mutually exclusive, then $A \cap B = \emptyset$

Venn diagram:



$S = \{0, 1, 2, 3, 4, 5, 6\}$

Example 8. For the experiment in which the number of pumps in use at a single six-pump gas station is observed, let $A = \{0,\ 1,\ 2,\ 3,\ 4\}$, $B = \{3,\ 4,\ 5,\ 6\}$, and $C = \{1,\ 3,\ 5\}$. Then

$$A^c = \{5, 6\}$$

$$A \cup B = \{0, 1, 2, 3, 4, 5, 6\}$$

$$A \cup C = \{0, 1, 2, 3, 4, 5\}$$

$$A \cap B = \{3, 4\}$$

$$A \cap C = \{1, 3\}$$

$$(A \cap C)^c = \{0, 2, 4, 5, 6\}$$

2.2 Axioms, interpretations, and Properties of Probability

Interpreting Probability

Consider an experiment that can be ____

and let A be an event consisting of a set of outcomes of the experiment. For examples,

the coin-tossing experiment previously discussed. If the experiment is performed ______________,

let __________ denote the number times on which A occurs. Then the ratio __________

is called ______________________________ of the event A in the sequence of n replications.

Given an experiment, the objective of probability is to ___________________________________,