

## Math 1070 - Spring '11 - Midterm 2

Name: \_\_\_\_\_ Date: \_\_\_\_\_

- In a test of hypotheses, the  $P$ -value is
  - the probability, assuming the null hypothesis is true, that the test statistic will take a value at least as extreme as that actually observed.
  - the probability, assuming the null hypothesis is false, that the test statistic will take a value at least as extreme as that actually observed.
  - the probability the null hypothesis is true.
  - the probability the null hypothesis is false.
- Two variables in a study are said to be confounded if
  - one cannot separate their effects on a response variable.
  - they are highly correlated.
  - they do not have a normal distribution.
  - one of them is a placebo.
- The law of large numbers states that as the number of observations drawn at random from a population with finite mean  $\mu$  increases, the mean  $\bar{x}$  of the observed values
  - gets larger and larger.
  - gets smaller and smaller.
  - tends to get closer and closer to the population mean  $\mu$ .
  - fluctuates steadily between one standard deviation above and one standard deviation below the mean.
- In their advertisements, the manufacturers of a certain brand of cereal would like to claim that eating their oatmeal for breakfast daily will produce a mean decrease in cholesterol of more than 10 mg/dl in one month for people with cholesterol levels over 200 mg/dl. In order to determine if this is a valid claim, they hire an independent testing agency, which then randomly selects 25 people with a cholesterol level over 200 for inclusion in a study. The 25 people in the study eat this cereal for breakfast daily for a month. Which of the following is the alternative hypothesis of interest in this study?
  - $H_a: \mu < 10$
  - $H_a: \mu > 10$
  - $H_a: \mu \neq 10$
  - $H_a: \bar{x} > 10$

5. A researcher selects a random sample. A 90% confidence interval for a population mean  $\mu$
- A) is an interval with margin of error  $\pm 90\%$ .
  - B) has the property that if we repeatedly selected our random sample in exactly the same way, each time constructing a different 90% confidence interval for  $\mu$ , then in the long run 90% of those intervals would contain  $\mu$ .
  - C) (a) and (b) are both true.
  - D) is an interval that has width .90.
6. Sickle-cell disease is a painful disorder of the red blood cells that in the United States affects mostly blacks. To investigate whether the drug hydroxyurea can reduce the pain associated with sickle-cell disease, a study by NIH gave the drug to 150 sickle-cell sufferers and the placebo to another 150. Neither doctors nor patients were told who received the drug. The number of episodes of pain reported by each subject was recorded. This is an example of
- A) an observational study.
  - B) an experiment, but not a double-blind experiment.
  - C) a double-blind experiment.
  - D) a paired data experiment.
7. Eggs that are contaminated with salmonella can cause food poisoning among consumers. A large egg producer takes an SRS of 200 eggs from all the eggs shipped in one day. The laboratory reports that 11 of these eggs had salmonella contamination. Unknown to the producer, 0.2% (two-tenths of one percent) of all eggs shipped had salmonella. In this situation
- A) 0.2% is a parameter and 11 is a statistic.
  - B) 11 is a parameter and 0.2% is a statistic.
  - C) both 0.2% and 11 are statistics.
  - D) both 0.2 % and 11 are parameters.
8. The high temperature (in degrees Fahrenheit) on January days in Columbus, Ohio varies according to the Normal distribution with mean 21 and standard deviation 10. If a January day in Columbus is randomly selected, what is the probability that the high temperature is between 15 and 25 degrees?
- A) .2743
  - B) .6554
  - C) .3811
  - D) almost 1

9. Suppose you decide to bet on red on each of 10 consecutive spins of the roulette wheel described in problem 7, above. Suppose you lose all 5 of the first wagers, which means that on the first 5 spins of the wheel, red has not come up once. Which of the following is true?
- A) You should get more spins of red on the next 5 spins of the wheel, since we didn't get any on the first 5 spins.
  - B) The wheel is not working properly. It favors outcomes that are not red. Hence, during the next five spins of the wheel, we're likely to continue to see few red outcomes.
  - C) We're due for a win, so the sixth spin of the wheel is very likely to come up red.
  - D) What happened on the first 5 spins tells us nothing about what will happen on the next 5 spins.
10. You randomly select 500 students and observe that 85 of them smoke. Estimate the probability that a randomly selected student smokes.
- A) 0.27
  - B) .50, since there are two possible outcomes for every student surveyed (smoke, don't smoke)
  - C) 0.17
  - D) 1.2
11. In a statistical test of hypotheses, we say the data are statistically significant at level  $\alpha$  if
- A)  $\alpha = 0.05$ .
  - B)  $\alpha$  is small.
  - C) the  $P$ -value is larger than  $\alpha$ .
  - D) the  $P$ -value is less than  $\alpha$ .
12. A study attempts to determine whether a football filled with helium when kicked travels farther than one that is filled with air. Each subject kicks twice—once with a football filled with helium and once with a football filled with air. The order of the type of football kicked is randomized. This is an example of
- A) a matched pairs experiment.
  - B) a double-blind observational study.
  - C) a stratified analysis.
  - D) the placebo effect.

13. The average age of residents in a large residential retirement community is 69 years with standard deviation 5.8 years. A simple random sample of 100 residents is to be selected, and the sample mean age  $\bar{x}$  of these residents is to be computed. We know the random variable  $\bar{x}$  has approximately a normal distribution because of
- A) the central limit theorem.
  - B) the law of large numbers.
  - C) the 68–95–99.7 rule.
  - D) the population we're sampling from has a Normal distribution.

- Sampling distribution of a sample mean:
  - $\bar{x}$  has mean  $\mu$  and standard deviation  $\sigma/\sqrt{n}$ .
  - $\bar{x}$  has a Normal distribution if the population distribution is Normal.
  - Central limit theorem:  $\bar{x}$  is approximately Normal when  $n$  is large.

## Basics of Inference

- $z$  confidence interval for a population mean ( $\sigma$  known, SRS from Normal population):

$$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}} \quad z^* \text{ from } N(0, 1)$$

- Sample size for desired margin of error  $m$ :

$$n = \left( \frac{z^* \sigma}{m} \right)^2$$

- $z$  test statistic for  $H_0 : \mu = \mu_0$  ( $\sigma$  known, SRS from Normal population):

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \quad P\text{-values from } N(0, 1)$$

## Inference About Means

- $t$  confidence interval for a population mean (SRS from Normal population):

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}} \quad t^* \text{ from } t(n-1)$$

- $t$  test statistic for  $H_0 : \mu = \mu_0$  (SRS from Normal population):

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \quad P\text{-values from } t(n-1)$$

- Matched pairs: To compare the responses to the two treatments, apply the one-sample  $t$  procedures to the observed differences.
- Two-sample  $t$  confidence interval for  $\mu_1 - \mu_2$  (independent SRSs from Normal populations):

$$(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

with conservative  $t^*$  from  $t$  with df the smaller of  $n_1 - 1$  and  $n_2 - 1$  (or use software).

- Two-sample  $t$  test statistic for  $H_0 : \mu_1 = \mu_2$  (independent SRSs from Normal populations):

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

with conservative  $P$ -values from  $t$  with df the smaller of  $n_1 - 1$  and  $n_2 - 1$  (or use software).

## Inference About Proportions

- Sampling distribution of a sample proportion: when the population and the sample size are both large and  $p$  is not close to 0 or 1,  $\hat{p}$  is approximately Normal with mean  $p$  and standard deviation  $\sqrt{p(1-p)/n}$ .

- Large-sample  $z$  confidence interval for  $p$ :

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad z^* \text{ from } N(0, 1)$$

Plus four to greatly improve accuracy: use the same formula after adding 2 successes and two failures to the data.

- $z$  test statistic for  $H_0 : p = p_0$  (large SRS):

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \quad P\text{-values from } N(0, 1)$$

- Sample size for desired margin of error  $m$ :

$$n = \left( \frac{z^*}{m} \right)^2 p^*(1-p^*)$$

where  $p^*$  is a guessed value for  $p$  or  $p^* = 0.5$ .



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**TABLE C** *t* distribution critical values

degrees of freedom	Confidence level <i>C</i>											
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
<i>z</i> *	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
One-sided <i>P</i>	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
Two-sided <i>P</i>	.50	.40	.30	.20	.10	.05	.04	.02	.01	.005	.002	.001

Math 1070 - Spring '11 - Midterm 2

Name: \_\_\_\_\_ Date: \_\_\_\_\_

A  B  C  D

1.  A  B  C  D

2.  A  B  C  D

3.  A  B  C  D

4.  A  B  C  D

5.  A  B  C  D

6.  A  B  C  D

7.  A  B  C  D

8.  A  B  C  D

9.  A  B  C  D

10.  A  B  C  D

11.  A  B  C  D

12.  A  B  C  D

13.  A  B  C  D

14. A manufacturing process produces bags of cookies. The distribution of content weights of these bags is Normal with mean 15.0 oz and standard deviation 1.0 oz. If 100 bags of cookies are selected randomly, what is the probability that the sample mean will be between 14.9 and 15.1 ounces?

15. A psychologist has developed a set of activities which she hopes will help children develop better reading skills. In a study of the effectiveness of these activities, one class of second grade children learns with the activities. Another class of second grade children serves as the control, and learned without the activities. After some period of time, the reading skills of all of these children were assessed. A summary of these data is:

	$n$	$\bar{x}$	$s$
Activities class:	21	51.48	11.01
No Activities class:	23	41.52	17.15

State the hypotheses, compute the statistic, find the P-value, and conclude. (use the conservative method for degrees of freedom)

16. A special diet is intended to reduce the cholesterol of patients at risk of heart disease. If the diet is effective, the target is to have the average cholesterol of this group be below 200. After six months on the diet, an SRS of 50 patients at risk for heart disease had an average cholesterol of  $\bar{x} = 192$ , with standard deviation  $s = 21$ . Is this sufficient evidence that the diet is effective in meeting the target? (State the hypotheses, compute the statistic, find the P-value, and conclude.)

17. The water diet requires one to drink two cups of water every half hour from when one gets up until one goes to bed, but otherwise allows one to eat whatever one likes. Four adult volunteers agree to test the diet. They are weighed prior to beginning the diet and after six weeks on the diet. The weights (in pounds) are

<u>Person</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
Weight before the diet	180	125	240	150
Weight after six weeks	170	130	215	152

For the population of all adults, assume that the weight loss after six weeks on the diet (weight before beginning the diet minus weight after six weeks on the diet) is Normally distributed with mean  $\mu$ . Find a 95% confidence interval for  $\mu$  based on these data. **To help speed up the computation, the standard deviation of the differences is  $s=13.64$ .**

18. A local board of education want to conduct a survey of residents in the community concerning a property tax levy on the coming local ballot. How large a sample  $n$  would they need to estimate *the proportion of residents in the community that support the property tax levy* with margin of error 0.04 with 95% confidence? Assume that you don't know anything about the value of  $p$ .
19. An inspector inspects large truckloads of potatoes to determine the proportion  $p$  in the shipment with major defects prior to using the potatoes to make potato chips. Unless there is clear evidence that this proportion,  $p$ , is less than 0.10, he will reject the shipment. He will test the hypotheses  $H_0: p = 0.10$ ,  $H_a: p < 0.10$ . He selects an SRS of 100 potatoes from the over 2000 potatoes on the truck. Suppose that 6 of the potatoes sampled are found to have major defects. What is the  $P$ -value of this test? What is the conclusion?
20. The time (in number of days) until maturity of a certain variety of tomato plant is Normally distributed with mean  $\mu$  and standard deviation  $\sigma = 2.4$ . A simple random sample of four plants of this variety is selected. The number of days until maturity for each plant is given below  
63 69 62 66

Based on these data, find a 99% confidence interval for  $\mu$ , in days.

## Answer Key - Midterm2-S11

1. A
2. A
3. C
4. B
5. B
6. C
7. A
8. C
9. D
10. C
11. D
12. A
13. A
14. 0.6826.
15. between 0.01 and 0.02.
16. below 0.01.
17.  $7 \pm 21.70$ .
18.  $n = 601$
19. 0.0918.
20.  $65.00 \pm 3.09$ .