Math 1310

Review Session for the First Midterm

1. Sketch the graph of a function satisfying the following properties:
   - the domain D is all the real numbers;
   - the range R is $[-1, 0) \cup (0, 1]$, i.e. all the numbers between -1 (included) and 1 (included), but with 0 excluded;
   - $\lim_{x \to +\infty} f(x) = 0$;
   - $\lim_{x \to -\infty} f(x) = 0$;
   - $\lim_{x \to 0} f(x) = \frac{1}{2}$, but f is not continuous at 0;
   - $f'(1) = 0$ and $f''(1) > 0$;
   - $f'(-1) > 0$ and $f''(-1) < 0$.
2. Compute the following limits
   - $\lim_{x \to 0} \frac{4x^3+2x}{x^2+x}$
   - $\lim_{x \to -1} \frac{x^2-1}{x^3+1}$
   - $\lim_{h \to 0} \frac{e^{x+h}-e^x}{h}$
   - $\lim_{x \to 1} \frac{x^2-1}{x^3-2x^2+x}$
3. The hyperbolic sine is defined as $\sinh(x) = \frac{e^x-e^{-x}}{2}$ and the hyperbolic cosine is defined as $\cosh(x) = \frac{e^x+e^{-x}}{2}$. Compute their derivatives.
4. Sketch the graph of the function $\frac{x^2}{1+x^2}$. To this aim, determine the domain, make a guess of the range, compute $f'(x)$ and $f''(x)$, find all critical values and inflection points.
5. Prove the reciprocal rule for derivatives

$$\left(\frac{1}{g}(x)\right)' = -\frac{g'(x)}{g^2(x)}$$

and use it, together with the product rule, to prove the quotient rule.