

Math 1310 Lab 2.

Name/Unid: _____ Lab section: _____

1. **(Circular motion)(See also example 3, 4 of Sec 1.7)**

Consider the following three parametric curves:

(1) $x = \cos \pi t, y = \sin \pi t, 0 \leq t \leq 2$

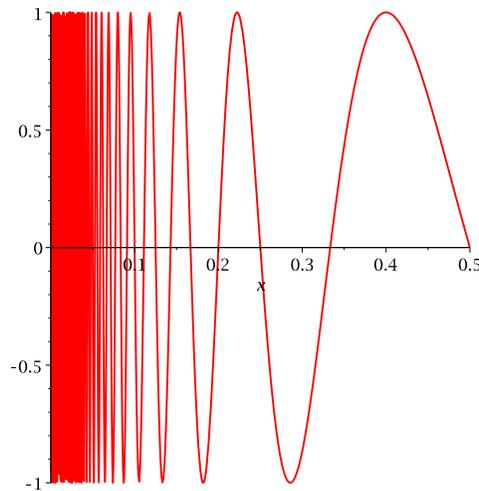
(2) $x = \cos 2\pi t, y = \sin 2\pi t, 0 \leq t \leq 1$

(3) $x = \cos(\pi t^2), y = \sin(\pi t^2), 0 \leq t \leq \sqrt{2}$

- (a) Plot these curves on a plane. Use a cartesian plane for each curve. **(3 pts)**
- (b) If we think of the variable t as time, the variables x, y being x, y coordinate of a particle, respectively, then the parametric curves above are describing the circular motion of that particle. Which particle is moving faster, the one in (1) or the one in (2)? **(1 pt)**
- (c) Are these particles moving clockwise or counterclockwise? **(1 pt)**
- (d) What particles in (1), (2), and (3) are moving in a constant speed? **(1 pt)**
- (e) Recall in Section 2.1, the average velocity of a moving object is defined by *ave. velocity = change in position/time elapsed*. Now we consider the moving particle in (1). If the unit of x, y are meters, and the unit of t is second, what is the average velocity of this particle from time 0 to time $1/2$? And from time 0 to time 1? (Don't forget to include the units to your answer) **(2 pts)**

Solution: A1. Unit circles. A2. (b) is faster. A3. counterclockwise. A4. (a)(b).
A5. (1) $\frac{\sqrt{2}}{1/2} = 2\sqrt{2}$ (m/sec) (2) 2 (m/sec).

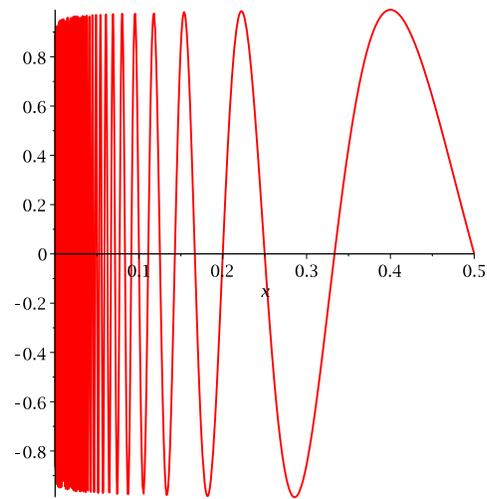
2. **(An oscillating function)**(See also example 4 of Sec 2.2) The following figure is the graph for $y = \sin(\pi/x)$. As you can see, when x is getting close to 0, y fluctuates more rapidly. It SEEMS that the function y of x does not have a limit at 0. We'd like to show our guess is true.



- (a) By plugging in x variable with values $1/2, 1/4, 1/6, 1/8, \dots$, what are the corresponding y values? **(1 pt)**
- (b) By plugging in x variable with values $2/3, 2/7, 2/11, 2/15, \dots$, what are the corresponding y values? **(1 pt)**

Therefore, by the observations in (a), y seems to approach to a limit when x is close to 0. By the observations in (b), y seems to have another limit when x is close to 0. Since we could only have one limit value, this suggests a contradiction - the limit at 0 does not exist.

Some of you might ask: Why do you make things so complicated? It seems quite obvious from the figure that the limit doesn't exist at 0. The problem is that the figure may be not precise enough to show you the true behavior near $x = 0$, so what you think to be OBVIOUS might turn out to be false. For example, consider the function $y = x^{1/100} \sin(\pi/x)$ (the graph is shown in the next page). This function has a limit at $x = 0$! (When you have learned the squeeze principle (Theorem 3, Sec 2.3) you'll know how to show the limit at $x = 0$ exists (and the limit is 0)).



Solution: A1. 0. A2. -1.

3. **(The limit of Gauss floor function)**(See also example 2, 3 of Sec 2.4)

The Gauss floor function $[x]$ means the greatest integer that is less than or equal to x . For example, $[2.3] = 2$, $[\pi] = 3$, $[9] = 9$.

- (a) Compute $[0]$ and $[-1.2]$. **(1 pt)**
- (b) Check wheter the limit $\lim_{x \rightarrow 7.3} [x]$ exists. If it does, compute it; otherwise provide an explanation why it does not exist. **(2 pts)**
- (c) Check wheter the limit $\lim_{x \rightarrow 7} [x]$ exists. If it does, compute it; otherwise provide an explanation why it does not exist. **(2 pts)**
- (d) Check wheter the limit $\lim_{x \rightarrow 2} [x^2]$ exists. If it does, compute it; otherwise provide an explanation why it does not exist. **(2 pts)**
- (e) Check wheter the limit $\lim_{x \rightarrow 1.5} [x]/x$ exists. If it does, compute it; otherwise provide an explanation why it does not exist. **(2 pts)**

Solution: A1. $[0]=0$, $[-1.2] = -2$. A2. 7, since the Gauss function is continuous at 7.3 (sketch the graph to see it). A3. Does not exist, since the left limit is 6 and the right limit is 7. A4. Does not exist, since the left limit is 3 and the right limit is 4. A5. $1/1.5 = \frac{2}{3}$, since we can use rules at page 104 and use the fact that the Gauss function has limit at 1.5.

4. **(Compute some inverse functions)(See also Sec 1.6)**

We know that given a function f , we can determine its domain D (i.e. where the function makes sense) and its range R (i.e. the set of values taken by the function). If a function f is injective (i.e. it assigns different values to different elements in the domain), we can find a function g which is the inverse of f . The domain of g will be the range of f and the range of g will be the domain of f . To determine whether a function is injective, there are a few standard techniques. One is the horizontal line test on the graph (try with $1/(x-1)$ to see it). Another one is to remember that the composition of two injective functions is still injective (for instance $\ln(x^3)$ is injective, since both $\ln(x)$ and x^3 are injective).

Determine the domain and the range of the following functions. Also, for each of those find the inverse $f^{-1}(x)$, and indicate a valid domain and range of $f^{-1}(x)$.

(a) $f(x) = \frac{x-1}{x+1}$ (2 pts)

(b) $f(x) = \ln(x^3 + 1)$ (2 pts)

(c) $f(x) = \frac{1}{1+e^{-x}}$ (2 pts)

Solution:

(a) The function is injective, since it passes the horizontal line test. The domain of f is $(-\infty, -1) \cup (-1, +\infty)$.

$$f = \frac{x-1}{x+1} \tag{1}$$

$$(x+1)f = x-1 \tag{2}$$

$$xf - x = -f - 1 \tag{3}$$

$$x(f-1) = -f-1 \tag{4}$$

$$x = (f+1)/(1-f) \tag{5}$$

The inverse function is $f^{-1}(x) = \frac{1+x}{1-x}$. The domain of f^{-1} is $(-\infty, 1) \cup (1, +\infty)$. The range is $(-\infty, -1) \cup (-1, +\infty)$.

(b) The function is injective since it is the composition of two injective functions ($\ln(x)$ and $x^3 + 1$). The domain of f is $(-1, +\infty)$.

$$f = \ln(x^3 + 1) \quad (6)$$

$$e^f = x^3 + 1 \quad (7)$$

$$x = (e^f - 1)^{1/3} \quad (8)$$

The inverse function is $f^{-1}(x) = \sqrt[3]{e^x - 1}$. The domain is all reals. The range is $(-1, +\infty)$.

(c) The function is injective since it is the composition of two injective functions ($\frac{1}{1+x}$ and e^{-x}). The domain of f is all reals.

$$f = \frac{1}{1 + e^{-x}} \quad (9)$$

$$(1 + e^{-x})f = 1 \quad (10)$$

$$f + e^{-x}f = 1 \quad (11)$$

$$e^{-x}f = 1 - f \quad (12)$$

$$e^{-x} = \frac{1 - f}{f} \quad (13)$$

$$e^x = \frac{f}{1 - f} \quad (14)$$

$$x = \ln\left(\frac{f}{1 - f}\right) \quad (15)$$

The inverse function is $f^{-1}(x) = \ln\left(\frac{x}{1-x}\right)$. The domain of f^{-1} is $(0, 1)$. The range is all reals.