

Grading and remarks for Lab 6

I did not take any points away if you make a clerical error, but I have corrected it for you, and you should pay some attention to these error.

If your solution is complete false, but you did something correct which help to solve the problem more or less, then I gave you 0.5 points. And I took 0.5 point away for the some small mistake which is not clerical error.

1. Most of students got the correct answer for these questions.

I took 1 point away for each error on calculating differentials, and for question 3, if you did not substitute $\sin^2 x$ by $1 - \cos^2 x$, then I also took 1 points away.

2. Most of students get the correct answer for this problem.

(Question 1) 1 point for $a'(b(c(x)))$, $b'(c(x))$ and $c'(x)$ respectively.

(Question 2) 0.5 point for $A'(x)$, 1 point for $B'(x)$ and 1.5 points for $f'(x) = A'(B(x))B'(x)$.

(Question 3) 1 point for $C'(x)$, $D'(x)$ and $f'(x) = C'(x)D(x) + C(x)D'(x)$ respectively.

(Question 4) took 1 point away if you did not replace y by $\frac{1}{(x^2+1)(x^2+2)}$ in the finally solution.

3. Change "10 hours" into "10 seconds" in question.

Lots of students have a trouble with this question, and some students got the same error: $A'(t) = -8\pi(1.00005)^2 e^{(t-10)}(1 - e^{(t-10)})$, they all miss the square in the solution.

(Question 1) $r'(t) = -A\alpha e^{(\alpha t - \beta)}$, which gives you the rate of change.

Since $r(0) = 1$, $r(10) = 0$ and $r'(0) = -\frac{e^{-10}}{1 - e^{-10}}$, then we have $r(0) = A(1 - e^{-\beta}) = 1$, $r(10) = A(1 - e^{10\alpha - \beta}) = 0$, $r'(0) = -A\alpha e^{-\beta} = -\frac{e^{-10}}{1 - e^{-10}}$.

The second equation $A(1 - e^{10\alpha - \beta}) = 0$ gives us $1 - e^{10\alpha - \beta} = 0$ since $A \neq 0$, then $10\alpha - \beta = 0$, i.e., $\beta = 10\alpha$.

By replacing β by 10α in the first and third equations, we get $A(1 - e^{-10\alpha}) = 1$, $-A\alpha e^{-10\alpha} = -\frac{e^{-10}}{1 - e^{-10}}$. The first one gives us $A = \frac{1}{1 - e^{-10\alpha}}$, plugging it into the second one, we get $-\frac{\alpha e^{-10\alpha}}{1 - e^{-10\alpha}} = -\frac{e^{-10}}{1 - e^{-10}}$, then simplify it, we obtain $\alpha = \frac{e^{10\alpha - 1}}{e^{10} - 1}$, obviously $\alpha = 1$ is a solution of this equation, and since $f(\alpha) = \alpha$ and $g(\alpha) = \frac{e^{10\alpha} - 1}{e^{10} - 1}$ are both strictly increasing functions, so they intersect in at most one point, thus $\alpha = 1$ is an unique solution.

Hence $\beta = 10\alpha = 10$ and $A = \frac{1}{1 - e^{-10\alpha}} = \frac{1}{1 - e^{-10}}$.

For this question, you can assume $\beta = 10$.

1 point for α , A and $r'(t)$ respectively.

(Question 2,3)

0.5 point for the area $S(t)$ (the volume $V(t)$) and 1.5 points for $S'(t)$ ($V'(t)$).