Math 1310 Lab 9.

Name/Unid: _____ Lab section: ____

1. **(Optimization)** Suppose that the energy needed for a swimming fish is $2 \times v^3$ Calories per second. If a fish is swimming upstream a distance of $100m$ with a current in the opposite direction of 3m/s, it will take the fish $100/(v-3)$ units of time to travel the distance of 100.

   (a) Write an expression for total energy expenditure of the fish as it travels upstream, $E(v)$. **(3 pts)**

   (b) Determine the velocity that minimizes energy used by the fish (assuming $v > 3$) **(3 pts)**

   (c) Graph $E(v)$ and label its minimum. **(2 pts)**

Solution:

(a) $E(v) = 2v^3 \dfrac{100}{v-3}$

(b) Derivative of $E(v)$ is $\dfrac{200v^2(2v-9)}{(v-3)^2}$, so $v = 9/2 = 4.5$, which is the minimum.

2. **(Optimization)** Suppose that an oil refinery is located on the north bank of a straight river that is 1 km wide. A pipeline is to be constructed from the refinery to the storage tanks located on the south bank of the river 5 km east of the refinery. The pipeline will first go east along the north bank of the river and then at point P the pipeline will go in a straight path south east to the storage tanks. If the coast of laying a pipe over land is \$300,000 per km and the cost of laying pipe over the river is \$500,000 per km, where should point P be to minimize cost? How much will it cost? **(5 pts)**

Solution: The cost for building the pipeline is (where x is the distance east P is from the refinery)

$$\begin{aligned} C(x) &= x * 300000 + \sqrt{1+(5-x)^2}(500000) \\ C'(x) &= 300000 - \frac{(5-x)}{(1+(5-x)^2)^{1/2}} * 500000 = 0 \\ x &= 17/4 = 4.25 \end{aligned}$$

This is the global minimum. Also the total cost with $x = 17/4$ is

$$C(17/4) = 1,900,000$$

3. **(Newton's Method)**

(a) Explain why Newton's Method doesn't work for finding $x^2 - 2x + 3 = 0$ with the initial point of $x_1 = 1$. **(4 pts)**

(b) Consider the equation $(x)^{1/5} = 0$. Use starting point $x_1 = 1$ to solve for zero with five subsequent iterations. Explain why Newton's Method is not working. **(4 pts)**

n	x_n	f(x_n)/f'(x_n)
1		
2		
3		
4		
5		
6		

(c) Use Newton's Method to fine $x^{1/5} = 2$ correct to 6 decimal places. **(4 pts)**

n	x_n	f(x_n)/f'(x_n)
1		
2		
3		
4		
5		
6		

Solution:

(a) Newton's Method doesn't work because the iterative process begins on a relative extremum so the slope at that point is zero.

(b) With Newton's method, we get the expression $x_{n+1} = -4x_n$. Starting with $x_1 = 1$, we find, $x_2 = -4, x_3 = 16, x_4 = -64, x_5 = 256, x_6 = -1024$. Newton's Method isn't working because the line approaches infinite slope near $x = 0$.

(c) Formulate the problem as $x^5 - 2 = 0$. We write Newton's Formula as $x_{n+1} = x_n - \dfrac{x_n^5 - 2}{5x^4}$. After some iterations with an initial point such as $x = 0$, we find $x = 1.148698$