

Math 1310 Lab 7.

Name/Unid: _____ Lab section: _____

Note: no credit is given for answers without any explanation.

1. **Deriving a trigonometric identity:** Show for all $-1 \leq x \leq 1$, we have $\arcsin(x) + \arccos(x) = \pi/2$ using the steps below.

(a) Show $\arcsin'(x) = \frac{1}{\sqrt{1-x^2}}$ by using the implicit derivative. **(3 pts)**

(b) Show $\arccos'(x) = -\frac{1}{\sqrt{1-x^2}}$ by using the implicit derivative. **(3 pts)**

(c) Let $f(x) = \arcsin(x) + \arccos(x)$ Show that $f'(x) = 0$ and hence $f(x)$ is a constant. What is $f(0)$? **(2 pts)**

(d) Use part (c) to get $\arcsin(x) + \arccos(x) = \pi/2$. Provide a brief geometric explanation related to the meaning of arcsin and arccos. **(3 pts)**

2. **Application of derivatives:** If a tank holds 5000 gallons of water, which drains from the bottom of the tank in 40 minutes, then Torricelli's Law gives the volume V of water remaining in the tank after t minutes as $V = 5000(1 - 0.025t)^2$, $0 \leq t \leq 40$ Find the rate at which water is draining from the tank after

(a) 10min (**1 pt**)

(b) 20min (**1 pt**)

(c) At what time is the water flowing out the fastest? The slowest? (**2 pts**)

3. **Application of derivatives:** If a stone is thrown vertically upward from the surface of the moon with a velocity of $10m/s$, its height h (in meters) after t seconds is given by the function $h = 10t - 0.8t^2$.

(a) What is the velocity of the stone after $3s$? **(1 pt)**

(b) What is the velocity of the stone after it has risen $25m$? **(1 pt)**

(c) Find the acceleration due to gravity near the surface of the moon. (Hint: the gravity is instantaneous change of the velocity of the stone before it hits the ground since no force other than the gravity is imposed on the stone once it's thrown into the sky.) **(2 pts)**

4. If two resistors, each with resistance $R_1(t)$ and $R_2(t)$ are connected to a circuit in parallel, the combined resistance R is given by

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}.$$

- (a) As the circuit heats up from use, the resistance of R_1 is changing at a rate of 0.4 Ohms per minute, and R_2 is changing at a rate of 0.2 Ohms per minute. What is the instantaneous rate of change of the combined resistor R when the two resistors have resistance value $R_1(t) = 10$ and $R_2(t) = 50$? **(3 pts)**
- (b) Now suppose the resistors are not changing over time and are set to have resistance value $R_1 = 10$ and $R_2 = 50$. However, the smaller resistor R_1 has some built-in variation, where the true value of the resistor is $R_1 = 10 \text{ Ohms} \pm 10\%$ due to manufacturer imperfections. Set $R_1 = 10+x$, and compute the linear approximation of the combined resistor R . What is the percent error of R given the variation in R_1 ? **(3 pts)**