Math 1310 Lab 6.

Name/Unid: ________________________________ Lab section: ____

Because of fall break, this lab is due on Friday, October 10th in class. No submission after the break will be accepted.

1. **(Warmup)**
   (a) Differentiate both sides of the identity $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ to obtain an identity for $\cos(2\theta)$ **(2 pts)**
   (b) Find the derivative of $f(x) = \sin(x)\sin(\frac{1}{x})$. By the squeeze theorem, the limit of this function exists when x approaches 0. Is its derivative defined when $x = 0$? **(3 pts)**

2. **(The Power Rule for fractional exponents)** The power rule for integer exponents can be proven from the limit definition of the derivative, but proving it for general exponents is tricky.

   For a rational number $q = \frac{n}{m}$ and positive real numbers x, the function $f(x) = x^{n/m}$ is defined to be the (unique) positive real number y such that $y^m = x^n$.

   Use implicit differentiation on the above identity and solve for y' to obtain the power rule for fractional exponents. (Hint: deduce from $y^m = x^n$ the identity $y^{-1} = x^{-\frac{n}{m}}$ and substitute the value of y^{-1} in the equation you get from implicit differentiation). **(5 pts)**

3. Consider the parametric curve $P(t)$ given by $x(t) = r\cos(2\pi t), y(t) = r\sin(2\pi t)$

   For every time t, we have $x(t)^2 + y(t)^2 = r^2$; that is to say, every point on the parametric curve lies on the circle of radius r

   If an object is following a parametric curve $x(t), y(t)$, the x and y components of its velocity are given by $x'(t)$ and $y'(t)$, and the line through the points $(x(t), y(t))$ and $(x(t) + x'(t), y(t) + y'(t))$ is tangent to the curve at the point $(x(t), y(t))$

   (a) Sketch the position and velocity of an object moving along $P(t)$ at $t = 1, 3/2$ and a few other times (at least two other times). **(5 pts)**

   (b) Give a physical interpretation of the quantities $x"(t)$ and $y"(t)$ and sketch them alongside position and velocity. **(5 pts)**

   (c) Now consider the parametric curve $P_1(t) = P(t^2)$. Sketch the position and velocity of an object moving along $P_1(t)$ (use at least four values of time for this task). (Hint: The chain rule may help you save some time redoing calculations.) **(5 pts)**