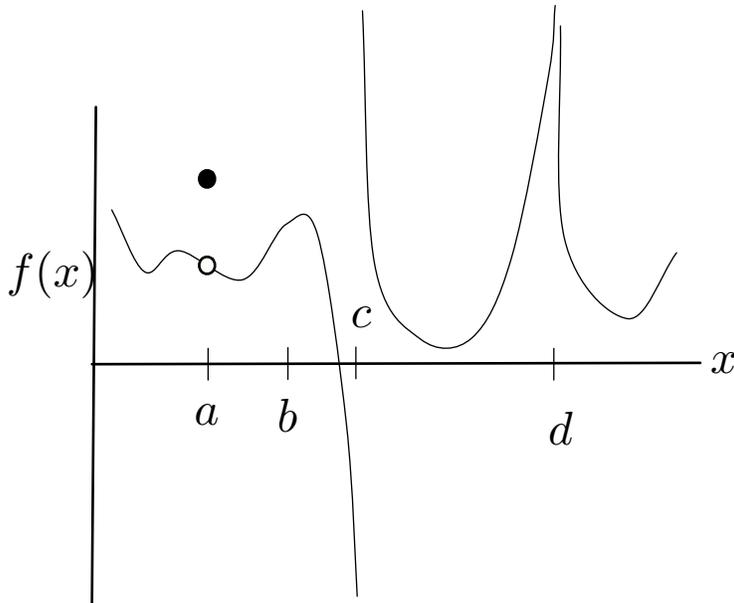


Math 1310 Lab 3.

Name/Unid: _____ Lab section: _____

1. For the graph indicated in the Figure, determine if the following limits exist. If any of the limits exists but is not finite, indicate whether it's positive infinity or negative infinity.



(a) $\lim_{x \rightarrow a} f(x)$ (1 pt)

(b) $\lim_{x \rightarrow b^-} f(x)$ (1 pt)

(c) $\lim_{x \rightarrow c} f(x)$ (1 pt)

(d) $\lim_{x \rightarrow c^+} f(x)$ (1 pt)

(e) $\lim_{x \rightarrow d} f(x)$ (1 pt)

(f) $\lim_{x \rightarrow d^-} f(x)$ (1 pt)

2. Using the definitions in the previous problem (i.e. if the value of the function should go to $+\infty$ or $-\infty$, we say that the limit exists and is equal to $+\infty$ or $-\infty$ respectively), compute the following limits if they exist. If any of the following limits does not exist, explain the reason. Hint: recall that $\cos 0 = 1$ and $\cos(x)$ is an even function.

(a) $\lim_{x \rightarrow 0^+} \frac{\cos(x)}{|x|}$ (1 pt)

(b) $\lim_{x \rightarrow 0^-} \frac{\cos(x)}{|x|}$ (1 pt)

$$(c) \lim_{x \rightarrow 0} \frac{\cos(x)}{|x|} \text{ (1 pt)}$$

$$(d) \lim_{x \rightarrow 0^+} \frac{\cos(x)}{x} \text{ (1 pt)}$$

$$(e) \lim_{x \rightarrow 0^-} \frac{\cos(x)}{x} \text{ (1 pt)}$$

$$(f) \lim_{x \rightarrow 0} \frac{\cos(x)}{x} \text{ (1 pt)}$$

3. In this exercise, the squeeze theorem is helpful. (The Squeeze Theorem says if $f(x) \leq g(x) \leq h(x)$ when x is near a (except possibly at a) and $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L$, then $\lim_{x \rightarrow a} g(x) = L$ where L can be $+\infty$ or $-\infty$. The conclusion holds also for one-sided limit.) Compute the following limits and answer relevant questions.

(a) $\lim_{x \rightarrow 0} x^2 \cos\left(\frac{1}{x}\right)$ **Hint: What are the $f(x)$ and $h(x)$ if you apply the squeeze theorem? (2 pts)**

(b) $\lim_{x \rightarrow 0^+} \frac{\sin(x)}{x}$ **Hint: for small positive x , $\sin(x) \leq x \leq \tan(x) = \frac{\sin(x)}{\cos(x)}$**

Transfer the inequalities into the form of $f(x) \leq \frac{\sin(x)}{x} \leq h(x)$, then apply the squeeze theorem. Again, what would be $f(x)$ and $h(x)$? (2 pts)

(c) $\lim_{x \rightarrow 0^-} \frac{\sin(x)}{x}$

Hint: it's related to part (b). Recall that $\sin(x)$ is an odd function which means $\sin(-x) = -\sin(x)$. (2 pts)

(d) Does $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$ exist? If so, what is it? If not, explain why it does not exist. (2 pts)

- (e) Can you explain the reason why for small positive x , $\sin(x) \leq x \leq \tan(x)$?
Hint: use a geometric argument on the unit circle. For more details, draw the unit circle in the coordinate system with x being a small positive angle in radian, what's the length of the small arc that corresponds to the angle? And what is the geometric object that has length $\sin(x)$ and $\tan(x)$ respectively? (2 pts)

4. Calculate the following values.

- (a) $\lim_{x \rightarrow 1} \frac{x^n - 1}{x - 1}$ Hint: Factor $x^n - 1$ into $(x - 1)g(x)$. What's $g(x)$? (1 pt)

(b) $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$ Hint: Use the method described in part (a) **(1 pt)**

(c) $\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2}$ Hint: Use the method described in part (a) **(1 pt)**