A STAB AT HOMOLOGICAL MIRROR SYMMETRY

FRANCO ROTA

ABSTRACT. These notes are an attempt at introducing some of the ideas around mirror symmetry and the homological mirror symmetry conjecture, first stated in [Kon95]. We start illustrating mirror symmetry at the level of Hodge numbers for the Fermat quintic. Then, following the survey [Bal08], we aim to illustrate a concrete example of homological mirror symmetry in the case of the projective line.

CONTENTS

1. INTRODUCTION

In the early '90s, physicists described a famous prediction on the number of rational curves on a quintic threefold in $\mathbb{P}^4$ [COGP92]. The computation was carried over by associating equivalent conformal field theories to a pair of Calabi-Yau manifolds. Since then, this duality between Calabi-Yau manifolds has been object of intense study and took the name of mirror symmetry. The name comes from the symmetry on the Hodge numbers of two mirror manifolds: if X and Y are a mirror pair then one observes:
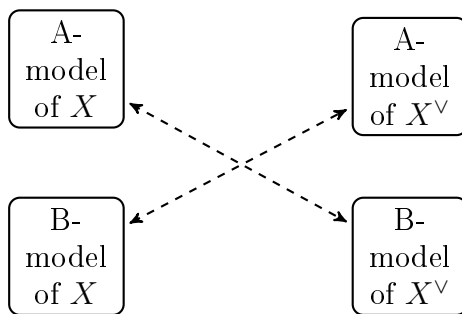
$$h^p(X, \Omega^q) = h^{n-p}(Y, \Omega^q).$$

The symmetry at a cohomological level soon inspired an interpretation of mirror symmetry as a correspondence of the complex and symplectic nature of the manifolds in the mirror pair, because of the interpretation one gives of the Hodge numbers $h^{1,2}$ and $h^{1,1}$.

Two main problems animated the study of this subject. The first is how to find mirror partners of a given complex manifold. Many mathematicians contributed to finding an answer in several different and elegant ways. An account on some of these constructions is [CR14], but we won't focus on this topic in these notes. Instead, we we illustrate the example of the Fermat quintic and its mirror, checking cohomological mirror symmetry.

The other problem is to package the symplectic data of the manifold (the A-model) and the complex data (the B-model) in a way that makes the correspondence clear. The following diagram describes the situation.

Date: October 17, 2017.

In 1994, Kontsevic proposed his Homological mirror symmetry conjecture [Kon95] as an attempt of explaining mirror symmetry from a categorical perspective. The idea is to interpret mirror symmetry as an equivalence of some triangulated categories, constructed using complex and symplectic geometry. In the second part of these notes we'll introduce some of the necessary terminology to phrase the conjecture, and explore the example of the projective line (following Ballard's survey [Bal08]).

2. Cohomological mirror symmetry for the Fermat quintic

In this section we compute the cohomology of the Fermat quintic hypersurface and that of its mirror, as constructed by Greene and Plesser in [GP90].

2.1. A useful result of Griffiths.
To compute cohomologies, we will make use of the following result of Griffiths [Gri69]. The exposition closely follows that of [CT99].

Consider a homogeneous polynomial P defining a smooth hypersurface X in $\mathbb{P}^{n+1}$. Let ν be a meromorphic differential on $\mathbb{P}^{n+1}$ with a pole of order $q+1$ along X. We can write $\nu = \nu(A, P, q)$ as

$$\nu(A, P, q) = \frac{A\Omega}{P^{q+1}}$$

where A is a homogeneous polynomial of degree $a(q) = (q+1) \deg P$, and Ω is the volume form $\Omega = \sum_i (-1)^i x_i dx_0 \wedge ... \wedge \widehat{dx_i} \wedge ... \wedge dx_{n+1}$. The form ν defines a cohomology class $\operatorname{res} \nu$ dually as follows:

$$\int_\gamma \operatorname{res} \nu = \frac{1}{2\pi} \int_{\partial T(\gamma)} \nu$$

where $T(\gamma)$ is a tubular neighborhood of the cycle γ. The classes $\operatorname{res} \nu$ where ν has a pole of order $q+1$ span precisely $F^{n-q} H_{\text{prim}}(X)$, the $(n-q)$-filtered piece of the primitive cohomology of X[1].

Since P is homogeneous, we can write

$$P(x_0, ..., x_{n+1}) = \frac{1}{\deg P} \left(\frac{\partial P}{\partial x_0}, ..., \frac{\partial P}{\partial x_{n+1}} \right) \cdot (x_0, ..., x_{n+1}),$$

therefore when A is a linear combination of the partial derivatives of P the residue class is cohomologous in $\mathbb{P}^n - X$ to a differential with a pole of order q. Let $J =$

---

[1] Recall that in complex projective geometry in $\mathbb{P}^n$, we have a specified Kähler class ω and a Lefschetz operator $L: H^i(X, \mathbb{C}) \to H^{i+2}(X, \mathbb{C})$ given by wedging with ω. We then define primitive cohomology $H_{\text{prim}}^{n-q}(X) = \ker(L^{q+1}: H^{nq}(X) \to H^{n+q+2}(X))$. One can compute the degree k primitive cohomology of X by studying the $k+1$th cohomology of $\mathbb{P}^n - X$ via a Gysin map. Since $\mathbb{P}^n - X$ is affine, this cohomology is spanned by rational residues (see [Gri69] for the details).

$(\frac{\partial P}{\partial x_0}, \ldots, \frac{\partial P}{\partial x_{n+1}})$, and denote by R the ring $\mathbb{C}[x_0, \ldots, x_{n+1}]/J$. Note that R is a graded ring and a finite dimensional $\mathbb{C}$-vector space. We have a map

$$\text{res} \colon R_{a(q)} \longrightarrow F^{n-q}H_{\text{prim}}(X) \Big/ F^{n-q+1}H_{\text{prim}}(X) \simeq H_{\text{prim}}^{n-q,q}(X).$$

Theorem 2.1 (Griffiths). *The map* res *is an isomorphism* $R_{a(q)} \simeq H_{\text{prim}}^{q,n-q}(X)$.

Remark 2.2. In particular, notice that if X is a Calabi-Yau threefold hypersurface then $H^3(X) = H_{\text{prim}}^3(X)$, since $H^5(X) = 0$ by Serre duality and the Lefschetz hyperplane theorem.

2.2. Cohomology of the Fermat quintic.
We now proceed to compute the Hodge diamond for the hypersurface $X = V(\sum x_i^5) \subset \mathbb{P}^4$, making use of Theorem 2.1. We will then describe the mirror to X, compute its Hodge numbers and compare the results.

The Hodge diamond for X is well known:

$$
\begin{array}{ccccc}
& & 1 & & \\
& & 1 & & \\
1 & 101 & & 101 & 1 \\
& & 1 & & \\
& & 1 & &
\end{array}
$$

The four vertical elements are give by the class of a hyperplane. One can compute $H^3(X)$ with Theorem 2.1: the elements of $H^{1,2}$ (we are in the case $q = 1$) are in bijection with elements $A \in R_{a(1)=5}$. These are monomials of degree 5 in 5 variables, with the condition that no variable appears with degree 4 or 5. There are

$$\binom{9}{4} - 20 - 5 = 101$$

elements in R_5 (all monomials of degree 5, minus the ones in which a variable has degree 4 and the ones in which one has degree 5). A similar computation counts 101 elements in R_{10}, while the only element of R_{15} is $\prod x_i^3$.

2.3. Cohomology of the mirror quintic.
The mirror of X was initially constructed by Greene and Plesser [GP90], who defined a family of mirror Calabi-Yau orbifolds. We will describe one element of the family. Start considering the quotient $\mathbb{P}^4/G$, where G is the group

$$G = \{(a_0, \ldots, a_4) \in \mathbb{Z}_5 \colon \sum a_i \cong 0 \bmod 5\}/\mathbb{Z}_5.$$

The copy of $\mathbb{Z}_5$ is embedded diagonally, and the group acts by multiplication of each of the variables by ζ^{a_i}, with $\zeta = e^{\frac{2\pi i}{5}}$. Then, consider the vanishing locus Y of $\sum_i x_i^5$ in $\mathbb{P}^4/G$.

Since Y is an orbifold, we will need a slightly more sophisticated way of computing its cohomology (and to make sense of it!). The best suited theory for our purposes is Chen-Ruan orbifold cohomology (see [CR04] for the definition, or Clader's expository notes [Cla]).

A precise exposition of the computation of the cohomology for the mirror quintic is carried over, for example, in [LS14]. Here we refrain from a thorough presentation of the technical parts of the computation, and focus on aspects that give a taste

of the combinatorics involved. The upshot is that for an element $g \in G$ we will consider the set Y^g of points of Y fixed by g. Then one has that the Chen-Ruan cohomology of Y, denoted $H^*_{\mathrm{CR}(Y)}$, is given by

$$H^*_{\mathrm{CR}}(Y) = \bigoplus_{g \in G} H^*(Y^g).$$

The degrees on the two sides of the equality don't match in general. One needs to take care of this aspect introducing *age shifts*, which are explained for example in Clader's notes [Cla]. If $g \neq \mathrm{Id}$ doesn't fix at least two coordinates x_i, one can see that $Y^g = \emptyset$. Otherwise, one can group elements in G according to $\sum a_i$. There are 100 elements fixing at least two variables and which satisfy $\sum a_i = 5$, for each of them Y^g is a point. One can count them as follows. If an element fixes two variables, it has the form $(0, 0, 1, 2, 2)$ or $(0, 0, 1, 1, 3)$ (up to permutations). Choose the two variables to be fixed, and the variable with a different exponent. There are $3\binom{5}{2}$ possible choices for each of the two forms. Likewise, if an element fixes three coordinates, the action on the other two is of the form $(1, 4)$ or $(2, 3)$. For each of the forms, we have $2\binom{5}{2}$ group elements. Then

$$2 \cdot 3 \binom{5}{2} + 2 \cdot 2 \binom{5}{3} = 100.$$

A similar reasoning for the case $\sum a_i = 10$ leads to the same number of elements.

When $g = \mathrm{Id}$, then $Y^g = Y$. In this case we argue differntly and use a more sophisticated version of Theorem 2.1. We can still look at the Jacobi ring R (defined exactly as above), but we incorporate the group action by only considering the invariant part R^G of the ring. There are exactly 4 polynomials, of degree 0,5,10 and 15 respectively, which are invariant under the action of G. These are generators for $H^3(Y)$. There are four other classes which are given by the hyperplane class in Y^g.

Putting all this together, one can see that the Hodge diamond of Y is

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & & 101 & & & \\
 & 1 & 1 & & 1 & 1 & \\
 & & & 101 & & & \\
 & & & 1 & & & \\
\end{array}
$$

This verifies mirror symmetry at a cohomological level for the Fermat quintic and its mirror.

3. Towards homological mirror symmetry

Once one has established mirror symmetry at a cohomological level, it is interesting to find a way to interpret this result. As string theory suggests, the interplay between complex and symplectic geometry should play a role in mirror symmetry. That would also explain the cohomological symmetry, since the Hodge numbers $h^{2,1}$ and $h^{1,1}$ of a Calabi-Yau threefold represent the dimension of the spaces of deformations of the complex, respectively symplectic, structure. Indeed, if X is a Calabi-Yau 3-fold we have $H^2(X, \Omega) \simeq H^1(X, \Omega^2) \simeq H^1(X, T_X)$, and the latter space parametrizes infinitesimal deformations of complex structures on X via the Kodaira-Spencer map (see [Voi02, Chp. 9] for a detailed explanation, intuitively, one can think of a cocycle valued in the tangent bundle as glueing data for the natural

complex structure on every bundle chart on which T_X is trivial). On the other hand, a symplectic form is a real $(1, 1)$ form, living naturally in $H^{1,1}(X) \cap H^2(X, \mathbb{R})$ (see [Huy06a, Sec. 3.1]).

In these notes, we will refrain from talking about the deep and surprising results that relate mirror symmetry and enumerative geometry (Gromov-Witten theory), as well as the interpretations of the phenomenon from the point of view of cohomological quantum field theories (all this deserves books for its own).

Instead, we want to give an illustration of Kontsevich's Homological mirror symmetry conjecture presented in [Kon95]. Possibly, the symmetries at a cohomological level reflect some deeper structure at the level of the derived category of coherent sheaves: this is sometimes the case because one can often interpret the cohomology $H^*(X)$ as a "linearized" version of $D^b(X)$ via the maps

$$D^b(X) \xrightarrow{q} K(X) \xrightarrow{\mathrm{ch}} H^*(X),$$

where $K(X)$ denotes the Grothendieck group of coherent sheaves on X, q is the standard map $\mathcal{E} \mapsto \sum_i (-1)^i [\mathcal{H}^i(\mathcal{E})]$ and ch denotes the Chern character[2]. The conjecture then aims at the definition of another triangulated category on the mirror partner Y and of an equivalence of categories. This second triangulated category should encode the symplectic data of Y. A good candidate for this is the Fukaya category of Y [Fuk93]. We won't define the Fukaya category in these notes, and we will limit ourselves to the illustration of an example in which homological mirror symmetry can be seen in action. Before describing the example, we recal some terminology from the symplectic geometry realm. A reference for this is [MS98].

3.1. **Symplectic geometry.** Let M be a smooth manifold, and let ω be an anti-symmetric two form on M.

Definition 3.1. *We say that ω is a symplectic form if it's closed and non degenerate, i.e. $d\omega = 0$ and ω defines a non-degenerate pairing on vectors in $T_x M$ for all $x \in M$.*

Example 3.2.   • The prototypical example of a symplectic manifold is the total space of the cotangent bundle T^*X to any smooth manifold X. If x_i denote coordinates of X and y_i are bundle coordinates, then the form $\sum_i dx_i \wedge dy_i$ is a symplectic form on T^*X.
      • Any Kähler manifold has a symplectic structure induced by the Kähler form.

It turns out that locally all symplectic manifolds are isomorphic, by the following theorem:

Theorem 3.3 (Darboux). *Let (M, ω) be a $2n$ dimensional symplectic manifold. Then every point of M has a neighborhood U and a local system of coordinates on U, denoted $(x_1, ..., x_n, y_1, ..., y_n)$ such that $\omega_{|U} = \sum_i dx_i \wedge dy_i$.*

We will be interested in a particular class of submanifolds of a symplectic manifold, to define them we need to investigate further the structure of the tangent bundle. If ω is a non-degenerate two form on a vector space T, for any subspace S one can define the symplectic orthogonal $S^\perp = \{v \in T : \omega(s, v) = 0 \text{ for all } s \in S\}$. We say that the subspace S is *Lagrangian* if $S = S^\perp$. Given a symplectic manifold M, a submanifold S is Lagrangian if all its tangent spaces are Lagrangian subspaces of the tangent space of M.

---

[2]A good reference for derived categories of coherent sheaves is [Huy06b]

Example 3.4. When M is T^*X as in Example 3.2, then the zero section, as well as every fiber, are Lagrangian submanifolds.

4. An Example: the projective line

We illustrate here one side of homological mirror symmetry in the (relatively) simple case of $\mathbb{P}^1 = \mathbb{P}^1_{\mathbb{C}}$ and its mirror (following [Bal08]). Namely, we give the definition of the B-model of $\mathbb{P}^1$ and of the A-model of the mirror, and check that they match. The mirror to $\mathbb{P}^1$ is not a variety, but something more sophisticated. It is a family of Landau-Ginzburg models: for a parameter $q \in \mathbb{C}^*$ one defines a map

$$W \colon \mathbb{C}^* \to \mathbb{C}$$
$$z \mapsto z + \frac{q}{z}$$

A pair $(W, \mathbb{C}^*)$ defines a Landau-Ginzburg model (LG model for brevity). One usually extracts interesting information from a LG model by studying the critical locus of W and its relation with the ambient space.

Remark 4.1. This is not an example of Calabi-Yau mirror symmetry, but of mirror symmetry for Fano varieties. The correspondence between complex and symplectic geometry is still visible, but one has to adjust the categories considered in Kontsevich's conjecture. Even if the example is much simpler than the general theory, it exposes many interesting features already.

4.1. **B-branes on the projective line.** We start describing the B-model of $\mathbb{P}^1_{\mathbb{C}}$, which corresponds to the complex side of mirror symmetry. The term branes originates from string theory: branes are boundary conditions for an open string which propagates through spacetime. In this example, the category of B-branes is $D(\mathbb{P}^1)$, the bounded derived category of coherent sheaves on $\mathbb{P}^1$. The category $D(\mathbb{P}^1)$ can be described as a category of representations of a certain quiver, as we will briefly describe.

Definition 4.2. *An object E of a k-linear triangulated category $\mathcal{T}$ is called exceptional if $\mathrm{Hom}(E, E[p]) = 0$ when $p \neq 0$, and $\mathrm{Hom}(E, E) = k$.*

An exceptional collection in $\mathcal{T}$ is a sequence of exceptional objects $(E_0, ..., E_n)$ satisfying the semiorthogonality condition $\mathrm{Hom}(E_i, E_j[p]) = 0$ for all p when $i > j$.

An exceptional collection $(E_0, ..., E_n)$ is called strong if, in addition, $\mathrm{Hom}(E_i, E_j[p]) = 0$ for all i and j when $p \neq 0$.

Proposition 4.3. *The category $D(\mathbb{P}^1)$ admits a full strong exceptional collection $(\mathcal{O}, \mathcal{O}(1))$. In particular, $D(\mathbb{P}^1)$ is the smallest triangulated category containing $\mathcal{O}$ and $\mathcal{O}(1)$.*

Recall that a *tilting bundle* on a smooth projective scheme Y is a finite rank locally free sheaf $\mathcal{E}$ such that $\mathrm{Ext}^k(\mathcal{E}, \mathcal{E}) = 0$ for $k \geq 0$ and such that if $F \in D(Y)$ satisfies $\mathrm{Hom}(\mathcal{E}, F) = 0$ then $F = 0$. In the above example, the sum $\mathcal{E} := \mathcal{O} \oplus \mathcal{O}(1)$ is a tilting bundle. We can represent the algebra of endomorphisms of $\mathcal{E}$, $\mathrm{Hom}(\mathcal{E}, \mathcal{E})$, with a quiver Q:

Definition 4.4. *Given a quiver Q, a path is a sequence of arrows $a_n...a_1$ concatenated from right to left. One defines the* path algebra kQ *of Q as the algebra generated by all the paths in the quiver (including an idempotent path at each vertex). The operations are formal sum and concatenation of paths.*

Lemma 4.5. *We have an algebra isomorphism $\mathrm{Hom}(\mathcal{E}, \mathcal{E}) \simeq kQ$.*

Proof. The two idempotent paths at the vertices correspond to projections on the two summands. The two arrows correspond to the two maps $x, y \colon \mathcal{O} \to \mathcal{O}(1)$. $\qquad\square$

The quiver algebra plays a fundamental role in understanding the derived category:

Proposition 4.6 ([Bal08, Prop. 3.6])**.** *There is an equivalence of triangulated categories $D^b(\mathbb{P}^1) \simeq D^b(kQ\text{-mod})$, where the latter denotes the derived category of left kQ-modules.*

Proof. We only give a sketch of the proof here. There are details to fill in, especially about the dg nature of the objects in the proof[3].

The strategy is to define a functor from the category $I(\mathbb{P}^1)$ of bounded complexes of injective complexes with coherent cohomology, then have it descend to $D(\mathbb{P}^1)$. Let $I_{\mathcal{E}}$ be an injective resolution of $\mathcal{E}$. Consider the algebra $A = \mathrm{Hom}_{I(\mathbb{P}^1)}(I_{\mathcal{E}}, I_{\mathcal{E}})$. The functor $\mathrm{Hom}_{I(\mathbb{P}^1)}(I_{\mathcal{E}}, -)$ takes injective complexes to chain complexes of $\mathbb{C}$-vector spaces which admit a left dg-module structure over A by precomposition. This induces a functor $I(\mathbb{P}^1) \to D(A\text{-mod})$.

To check that this functor descends it is enough to make sure that null-homotopic complexes get sent to zero (quasi-isomorphisms are already invertible in $I(\mathbb{P}^1)$). But if N is a null-homotopic complex then its contracting homotopy induces a contracting homotopy for $\mathrm{Hom}_{I(\mathbb{P}^1)}(I_{\mathcal{E}}, N)$. Since $A \simeq kQ$, their derived categories are equivalent. Hence we get a functor

$$\mathrm{RHom}(\mathcal{E}, -) \colon D(\mathbb{P}^1) \to D(kQ)$$

The properties of a tilting bundle imply that $\mathrm{RHom}(\mathcal{E}, -)$ is fully faithful. It is also essentially surjective on $D(kQ)$. $\qquad\square$

4.2. **A-branes on the mirror to the projective line.** Recall that the mirror to $\mathbb{P}^1$ is the Landau-Ginzburg model $W \colon \mathbb{C}^* \to \mathbb{C}$, with $W(z) = z + \frac{1}{z}$. We start fixing a symplectic form on $\mathbb{C}^*$. We pick a symplectic form ω, for example $\omega = i\frac{dz \wedge d\bar{z}}{z\bar{z}}$.

We look for non-closed Lagrangian submanifolds whose boundary lies in the fiber $W^{-1}(t)$ of a generic point $t \in \mathbb{C}$. The singular points of W are $p \in \mathbb{C}^*$ such that $W'(p) = 0$. The singular points in this example are $p_1 = 1$ and $p_2 = -1$, and we denote the corresponding singular values by $q_i = W(p_i)$, $i = 1, 2$.

The fiber over a general point $t \in \mathbb{C}$ consists of two points (in general, the fiber is an m-dimensional sphere where m is the relative dimension of W). We then pick a path γ connecting t and a singular value q. The idea is now to flow the general fiber along γ to the critical point, so that it collapses to a point, yielding a $m + 1$-dimensional disc.

---

[3]For the aspects in homological algebra, in particular the topics of dg-algebras, A_∞-categories, and Hochschild homology, we refer the interested reader to the survey [Bal08] and to the references therein.

This can be done respecting the symplectic structure: the form ω allows us to split the tangent space of $\mathbb{C}^*$ into the tangent space of the fiber and its symplectic orthogonal (again, this decomposition is trivial here because W has relative dimension 0). We can then transport each point $z_0 \in W^{-1}(t_0)$ "horizontally", i.e. using the symplectomorphism coming from parallel transport along γ.

In particular, in our example we get intervals L_1 and L_2, one for each critical value q_1 and q_2, which are Lagrangian submanifolds with boundary in $W^{-1}(t_0)$. We refer to these Lagrangian submanifolds as *vanishing thimbles*.

Shortly we will define the category of vanishing cycles of W, whose objects are the vanishing thimbles. In general, the morphisms of such a category are defined using Floer cohomology and the Fukaya category (see [Bal08, Sec. 2.4]), we won't say much about this in these notes, but it's worth noticing that symplectic geometry plays a role in the definition of morphisms too.

We are now ready to give the definition of the category of A-branes for the mirror to $\mathbb{P}^1$:

Definition 4.7. *The category of vanishing cycles* $\mathrm{FS}(W)$ *for a LG model W is an A_∞-category whose objects are the vanishing thimbles L_1 and L_2. The morphisms are*

$$
(1) \qquad \mathrm{Hom}(L_i, L_j) = \begin{cases} \mathbb{C}^2 \text{ if } i < j; \\ \mathbb{C}\,\mathrm{Id}_{L_i} \text{ if } i = j; \\ 0 \text{ else.} \end{cases}
$$

Then, the category of A-branes on W is the bounded derived idempotent closure of the category of vanishing cycles, denoted $D^\pi(\mathrm{FS}(W))$.

Remark 4.8. (1) The two copies of $\mathbb{C}$ in $\mathrm{Hom}(L_1, L_2)$ are related to a count of holomorphic discs: there are two ways of mapping a holomorphic disc in $\mathbb{C}$ such that the upper half of its boundary maps to L_1 and the lower half to L_2. In general, a much more sophisticated version of this count defines morphisms in the Fukaya category.
   (2) The morphism algebra of this category is isomorphic to the path algebra of the quiver Q we studied before, and $D^\pi(\mathrm{FS}(W))$ is the category of finite-dimensional representations of Q.
   (3) In fact, there is an equivalence of triangulated categories $D^\pi(\mathrm{FS}(W)) \simeq D^b(\mathbb{P}^1)$. This verifies the homological mirror symmetry conjecture for $\mathbb{P}^1$.

References

[Bal08]   M. R. Ballard. "Meet homological mirror symmetry". In: *Modular forms and string duality*. Vol. 54. Fields Inst. Commun. Amer. Math. Soc., Providence, RI, 2008, pp. 191–224.

[Cla]   *Emily Clader's webpage.* https://sites.google.com/site/emilyclader/writing.

[COGP92]   P. Candelas, X. C. de la Ossa, P. S. Green, and L. Parkes. "An exactly soluble superconformal theory from a mirror pair of Calabi-Yau manifolds". In: *Particles, strings and cosmology (Boston, MA, 1991)*. World Sci. Publ., River Edge, NJ, 1992, pp. 667–680.

[CR04]   W. Chen and Y. Ruan. "A new cohomology theory of orbifold". In: *Comm. Math. Phys.* 248.1 (2004), pp. 1–31. ISSN: 0010-3616.

[CR14]   E. Clader and Y. Ruan. "Mirror Symmetry Constructions". In: *ArXiv e-prints* (2014). arXiv: `1412.1268 [math.AG]`.

[CT99]   J. A. Carlson and Domingo Toledo. "Discriminant complements and kernels of monodromy representations". In: *Duke Math. J.* 97.3 (1999), pp. 621–648. ISSN: 0012-7094.

[Fuk93]   Kenji Fukaya. "Morse homotopy, AˆâĹđ Category and Floer homologies". In: *Proceeding of Garc Workshop on Geometry and Topology.* Seoul National Univ. 1993.

[GP90]   B. R. Greene and M. R. Plesser. "Duality in Calabi-Yau moduli space". In: *Nuclear Phys. B* 338.1 (1990), pp. 15–37. ISSN: 0550-3213.

[Gri69]   P. A. Griffiths. "On the periods of certain rational integrals. I, II". In: *Ann. of Math. (2) 90 (1969), 460-495; ibid. (2)* 90 (1969), pp. 496–541. ISSN: 0003-486X.

[Huy06a]   D. Huybrechts. *Complex geometry: an introduction.* Springer Science & Business Media, 2006.

[Huy06b]   D. Huybrechts. *Fourier-Mukai transforms in algebraic geometry.* Oxford Univ. Press, 2006.

[Kon95]   Maxim Kontsevich. "Homological algebra of mirror symmetry". In: *Proceedings of the International Congress of Mathematicians, Vol. 1, 2 (Zürich, 1994).* Birkhäuser, Basel, 1995, pp. 120–139.

[LS14]   Y.-P. Lee and M. Shoemaker. "A mirror theorem for the mirror quintic". In: *Geom. Topol.* 18.3 (2014), pp. 1437–1483. ISSN: 1465-3060.

[MS98]   D. McDuff and D. Salamon. *Introduction to symplectic topology.* Oxford University Press, 1998.

[Voi02]   C. Voisin. *Hodge theory and complex algebraic geometry. I. Translated from the French original by Leila Schneps.* Vol. 76. 11. Cambridge Studies in Advanced Mathematics, 2002.

F. Rota, Department of Mathematics, University of Utah, Salt Lake City, UT 84112, USA

E-mail address: `rota@math.utah.edu`