Math 1310 Lab 8.
Name/Unid: _____________________________ Lab section: ___

Note: no credit is given for answers without any explanation.

1. (Related rates 1)

Two cars (car A and car B) take part in a race in the desert. The race consists of crossing the desert moving east along a straight line. Car A and car B start at the same time. Car A has an initial speed of 50\( \text{km/h} \) and has a constant acceleration of 10\( \text{km/h}^2 \). Car B moves with constant speed 60\( \text{km/h} \). Remember to include the units of measure in your answers to the following questions.

(a) At what rate is car A approaching car B, when the velocity of car A coincides with the velocity of car B? After having found this value with computation, explain in words its physical meaning. (3 pts)

(b) At what instant of time does car A overtake car B? (2 pts)

(c) The kinetic energy of an object is given by \( K = \frac{1}{2}mv^2 \), where \( m \) is the mass of the object and \( v \) is its speed. Assume the mass of car A is 1000\( \text{kg} \). How fast is the kinetic energy of car A increasing when car A and car B have the same speed? And when car A is overtaking car B? (3 pts)

Solution:

(a) We have the positions \( x_A = 50t + 5t^2 \) and \( x_B = 60t \) in function of the time (hours) and velocities \( v_A = 50 + 10t \) and \( v_B = 60 \). Then we set \( v_A = v_B \), which is \( 50 + 10t = 60 \). Hence after \( t = 1\text{hrs} \) the two speeds coincide. Then the distance is \( x_B - x_A = 10t - 5t^2 \). Its derivative is \( 10 - 10t \) and evaluating it in \( t = 1 \) we get 0. It is reasonable since if \( v_A = v_B \) at some instant of time \( t \), it means that neither car A is getting closer to car B, nor car B is running away from car A. (3 pts)

(b) We have to set \( x_A = x_B \), which means \( 10t - 5t^2 = 0 \). Since 0 is not a solution of our problem, the answer is \( t = 2\text{hrs} \). (2 pts)

(c) We have that the derivative to consider is \( mv_Av'_A = 10,000(50 + 10t) \) and we have to evaluate it in 1 and 2. (3 pts)
2. (Related rates 2)

In physics, the energy stored in a stretched spring is determined by the equation

\[ E = \frac{1}{2} k x^2, \]

where \( E \) is the energy, \( k \) is a constant (the \textit{spring constant}), and \( x \) represents the distance that the spring has been stretched.

(a) Find a formula for \( \frac{dE}{dt} \) in terms of \( k, x, \) and \( \frac{dx}{dt}. \) (3 pts)

(b) A spring with spring constant \( k = 0.20 \) Joules/cm\(^2\) is being stretched at a rate of 1.5 cm/sec. How quickly is the energy stored in the spring increasing at the moment that \( x = 10 \) cm? (3 pts)

\[ \text{Solution:} \]

(a) The formula is \( kx \frac{dx}{dt}. \)

(b) Using the formula above, we have \( 0.2 \cdot 10 \cdot 1.5 J/s = 3 J/s. \)
3. **(Optimization)** An industry producing milk has decided to renovate the milk box. For storage reasons, they have decided to give it the shape of a rectangular parallelepiped (i.e. a solid with rectangular shape) with squared base. The new milk box will have the volume of 1 liter (remember $1l = 1000cm^3$). What height should the milk box have to minimize the carton paper used? (6 pts)

**Solution:** We call the heigth $h$ and the side of the square $l$. Then we have $l^2h = 1000cm^3$. So we get $h = 1000/r^2$. We want to minimize the surface, which is $S(l) = 2l^2 + 4l \cdot \frac{1000}{l^2} = 2l^2 + \frac{4000}{l}$. We take the derivative, which is $S'(l) = 4l - \frac{4000}{l^2}$. We look for the critical values, i.e. $S'(l) = 0$. Then we get $4l - \frac{4000}{l^2} = 0$. By the physics of the problem, only $l > 0$ makes sense. So we multiply by $l^2$ and get $4l^3 = 4000$. The only solution is $l = 10cm$. It is a global minimum, since it is the only critical point and $S(l)$ has limit $+\infty$ both in 0 and $+\infty$. 
4. (Using l'Hospital’s Rule) Using l'Hospital’s Rule, evaluate the following limits.

(a) \( \lim_{x \to -1} \frac{x^2 - 1}{x + 1} \) (1 pt)

(b) \( \lim_{x \to +\infty} \frac{2x^2 + 4x - 1 - 1}{4x^2 - 5x + 3} \) (1 pt)

(c) \( \lim_{x \to +\infty} \frac{2e^x + 5}{6 - 4e^x} \) (1 pt)

(d) \( \lim_{x \to 0} \frac{\arctan(1 + x^2) - \frac{\pi}{4}}{\sin(x)} \) (1 pt)

(e) \( \lim_{x \to 0^+} \frac{\ln(\ln(x + 1))}{\ln(x)} \) (1 pt)

Solution:

(a) -2

(b) \( \frac{1}{2} \)

(c) \(-\frac{1}{2}\)

(d) 0

(e) 1

(f) \( \frac{1}{3} \)

(g) \( \pm \infty \)

(h) 0