Math 1310 Lab 5.
Name/Unid: __________________________ Lab section: ____

1. (Derivatives) Using the definition of the derivative, calculate the derivative of the following functions.
   (a) $3x^2 - 2x + 1$
   (b) $\sqrt{1 + 2x}$
   (c) $\frac{1}{x^2 - 2}$

Solution:
   (a) $6x - 2$
   (b) $\frac{1}{\sqrt{1 + 2x}}$
   (c) $\frac{-2x}{(x^2 - 2)^2}$
2. (Climate Change)

Suppose that an adjusted plot the average high temperature in New York City is 
\[ f(x) = 60 - 30 \cos \left( \frac{2\pi}{12} x \right) + \frac{x}{5}, \]
where \( x \) represents months beginning with January 2000.

(a) Find \( f'(x) \), and describe the physical meaning of \( f'(15) \) including its units.

(b) Calculate the average slope in the interval \( 0 \leq x \leq 24 \), and \( 6 \leq x \leq 18 \).

(c) Give an equation for the best fit linear approximation of \( f(x) \). Hint: Consider the average slope during an arbitrary 12-month cycle, and combine this with a y-intercept.

(d) What does the slope of your linear approximation mean? Remember to include units.

\[ \text{Solution:} \]

(a) \( f'(x) = \frac{2\pi}{12} \sin \left( \frac{2\pi}{12} x \right) + \frac{1}{5} \), and \( f'(18) = \frac{2\pi}{12} + \frac{1}{5} \), which means that in March 2011, the weather is increasing at \( \frac{2\pi}{12} + \frac{1}{5} \) degrees per month.

(b) It will be an average slope of \( \frac{1}{5} \) in both cases.

(c) \( g(x) = 60 + \frac{x}{5} \).

(d) The slope of \( \frac{1}{5} \) implies that New York City is warming at a rate of \( \frac{1}{5} \) of a degree per year.
3. (Bacterial Growth) The graph below, \( f(t) \) denotes the bacterial growth (units of thousands of bacteria per hour) in a petri dish.

(a) When is this function positive? When is this function increasing, and when is this function decreasing?

(b) Describe what is physically happening when \( t = 2 \). What could be an explanation for this change?

(c) Draw a graph of the amount of bacteria in the dish. Suppose that there are 1 thousand bacteria in the dish at time 0.

**Solution:**

(a) It is positive for all time. It is increasing when \( 0 \leq t > 2 \), and it is decreasing when \( 2 < t \).

(b) When \( t = 2 \), we see that the growth of bacteria is no longer getting larger. We reach a point of slowing growth. This slowing is probably due to a carrying capacity.

(c) At \( t = 2 \) there is an inflection point, and the graph should level out.
4. (Unemployment Rate) The following is a chart from the Bureau of Labor Statistics of the January unemployment rate since 2004.

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<th>Year</th>
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<tr>
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<td>2014</td>
<td>6.6</td>
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</tbody>
</table>

(a) Let $U(t)$ be the function of unemployment rate. What is the meaning of $U'(t)$, and what are the units?

(b) Construct a table of estimated values for $U'(t)$. Hint: you will have 10 data points for $U(t)$. When does $U'(t)$ go from being negative to positive? What does this mean, and what is it’s cause?

**Solution:**

(a) $U'(t)$ is the change in unemployment rate per year. Units are the percent of unemployment per year.


(c) Between 2006 and 2007 there is a change in slope from negative to positive, which means that the unemployment rate is no longer decreasing. The financial crisis caused the unemployment rate to stop decreasing and to jump 2.8 points the following year.
5. **(Functions are their Derivatives)** In the following graphs, determine which curves are the original function, and which are its first and second derivative.
Solution:
(a) Red is the original function. Blue is the first derivative. Green is the second derivative.
(b) Red is the original function. Green is the first derivative. Blue is the second derivative.
(c) Green is the original function. Red is the first derivative. Blue is the second derivative.