1. (Integration) Consider a vehicle on a road. It starts at velocity of 25 miles per hour \( v_0 = 25 \). At \( t = 1/2 \), the car gets on a freeway, and travels at 65 miles per hour \( v_1 = 65 \). Then at \( t = 3/4 \), the car leaves the freeway and travels on a road at 45 miles per hour \( v_2 = 45 \) until \( t = 1 \). Suppose the car starts at position \( x = 0 \).

(a) How far has the car gone at \( t = 1 \)? \( 2 \) pts

(b) When will the car have traveled 30 miles? \( 2 \) pts

(c) Create the position function of the car, \( x(t) \), from the velocity function and graph the function \( x(t) \). \( 3 \) pts
2. (Splitting and recombining)

Suppose $f$ and $g$ are two continuous functions

(a) If we know that

$$\int_0^2 (f + g)\,dx = 6, \quad \int_0^2 (3f + 3)\,dx = 12$$

Use linearity of the integral to find $\int_0^2 g\,dx$ (3 pts)

(b) If we know that

$$\int_0^3 (f + g)\,dx = 5, \quad \int_0^1 (f + g)\,dx = 2, \quad \text{AND } f(x) = 2 \text{ when } x \geq 1$$

Find $\int_1^3 g\,dx$

(Hint: Break up the integral using the property that $\int_a^c f\,dx = \int_a^b f\,dx + \int_b^c f\,dx$) (3 pts)
3. (Approximation)

If \( f \) is a continuous, decreasing function, the left-endpoint and right-endpoint Riemann sums \( L_n \) and \( R_n \) give upper and lower bounds for the integral;

\[
R_n \leq \int_a^b f \, dx \leq L_n
\]

There is a similar inequality if \( f \) is a continuous, increasing function.

(a) Compute the 5th left-endpoint and right-endpoint Riemann sums for the integral \( \int_2^1 x \, dx \). Use these to give bounds on the value of the integral. (3 pts)

(b) Compute the 5th left-endpoint and right-endpoint Riemann sums for the integral \( \int_1^0 t - t^2 \, dt \). Do these sums give bounds on the value of the integral? (3 pts)
4. (Antiderivatives..?) Recall that a function \( F \) is called an antiderivative for \( f \) if \( F' = f \).

Consider the function;

\[
f(t) = \begin{cases} 
1 & \text{if } t \leq 1 \\
2 & \text{if } t > 1
\end{cases}
\]

(a) Sketch the graph of \( F(x) = \int_0^x f(t)dt \). Is \( F(x) \) an antiderivative for \( f \)? What is \( F'(1) \)? (3 pts)

(b) What if we instead consider the following function;

\[
g(t) = \begin{cases} 
1 & \text{if } t \leq 1 \\
t & \text{if } t > 1
\end{cases}
\]

Sketch the graph of \( G(x) = \int_0^x g(t)dt \). Is this an antiderivative? What is \( G'(1) \)? (3 pts)