MATH 5620 – HOMEWORK #1
DUE WED FEB 1ST

1. (K&C 4.6.2) Show that if A has this property (unit row diagonally dominant):
$$a_{ii} = 1 > \sum_{j=1,\ j\neq i} |a_{ij}|,\ \text{for } 1 \leq i \leq n,$$
then Richardson iteration converges. **Hint:** Use the $\|\cdot\|_\infty$ induced matrix norm.
2. Please do parts (a) and (b) of the following problems.
   - B&F 9^{th} ed: 7.3.6, 7.3.8 and 7.4.6.
   - B&F 8^{th} ed: 7.3.6, 7.3.8 and 7.3.14.

   Please include the first couple of iterates and also the last one. In addition create a table with the number of iterations needed to the reach the specified tolerance, similar to

	Jacobi	Gauss-Seidel	SOR $\omega = 1.2$
(a)	x	x	x
(b)	x	x	x

   For this problem please stick with the algorithms of the class textbook. Do not use Matlab's backslash to solve the diagonal (or triangular) systems.
   Sample code is available in the class website (`prob2.m`, `jacobi.m`, `gs.m`, `sor.m`).
3. (Trefethen and Bau, §40)
   (a) Write the preconditioned conjugate gradient (CG) algorithm (see p32 of `math5620s12_s01.pdf` in class website). Be sure to test your code on a small system.
   Sample code (`mycg.m`) is available in the class website.
   (b) Let $A \in \mathbb{R}^{1000\times1000}$ be a symmetric matrix with $a_{ii} = \sqrt{i} + 0.5$ and $a_{ij} = 1$ when $|i-j| = 1$ or $|i-j| = 100$. Such a matrix can be generated in sparse form as follows:

   ```
   D = ones(1000,5); D(:,3) = sqrt(1:1000)+0.5;
   A = spdiags(D,[-100,-1,0,1,100],1000,1000);
   ```

   Produce a `semilogy` plot with the number of iterations in the abscissa and the residual in the ordinate comparing the convergence history for the unpreconditioned CG and the preconditioned CG for solving the system $Ax = b$ with initial guess $x = 0$ and
   $$b = (1, \ldots, 1)^T.$$
   For the preconditioner use Jacobi, i.e. the diagonal part of A. **Note:** You do not need to write the un-preconditioned CG as it corresponds to setting $M = I$ (identity) in the preconditioned code.