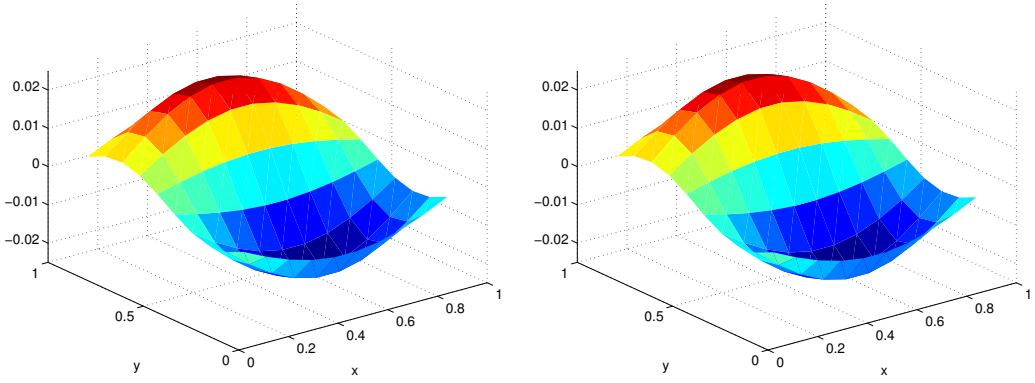
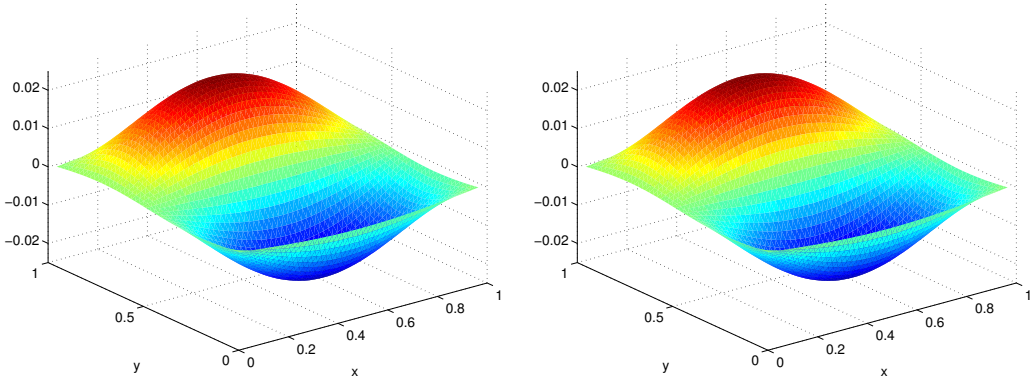
HW 4 Problem 1 solutions

true | calculated

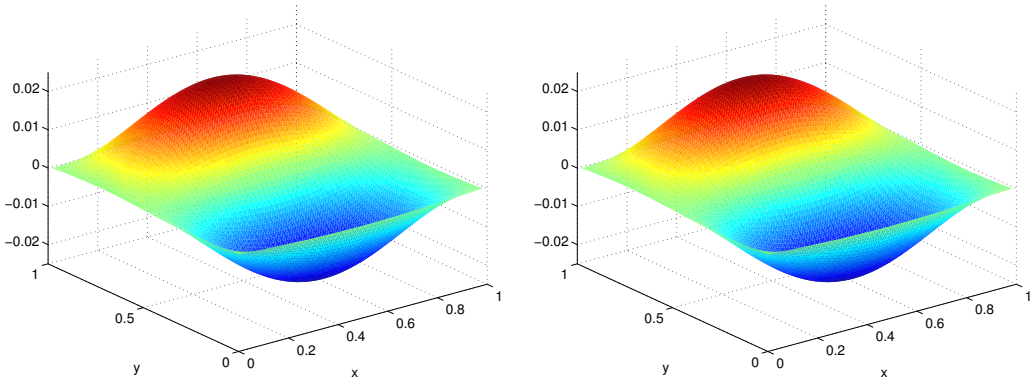
$n = 10$

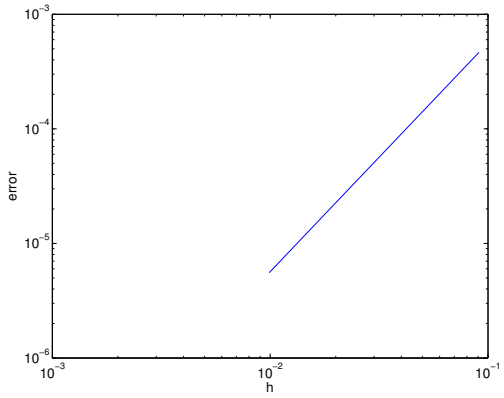


$n = 50$



$n = 100$





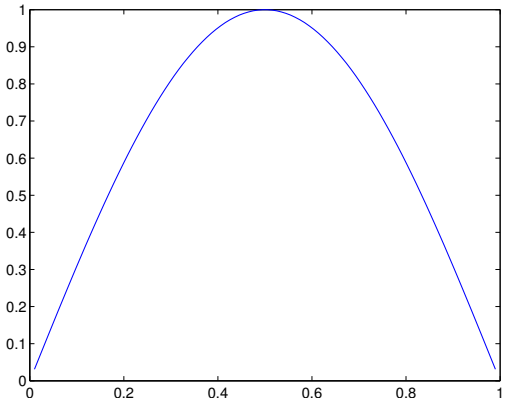
error

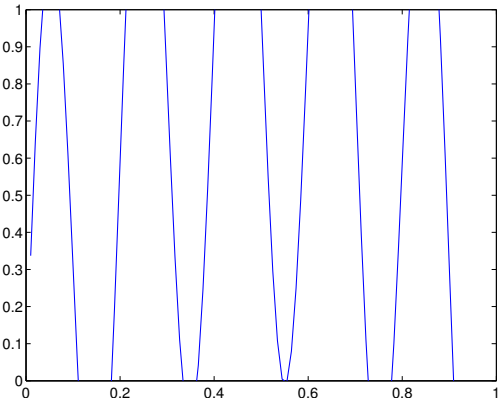
```
% Math 5620 HW 4 Problem 1
% Fernando Guevara Vasquez

ns = [10,50,100];
for i = 1:length(ns), n = ns(i);
 h = 1/(n+1);
 f = @(x,y) sin(pi*x).*sin(2*pi*y);
 utrue = @(x,y) -f(x,y)/5/pi^2;

 % get grid nodes
 x = linspace(0,1,n+2); y = linspace(0,1,n+2);
 % only the nodes that are not specified with boundary conditions
 [X,Y] = ndgrid(x(2:n+1),y(2:n+1));

 % form matrix
 I = eye(n); e = ones(n,1);
 T = spdiags([e -4*e e],-1:1,n,n);
 S = spdiags([e e], [-1,1],n,n);
 A = ( kron(I,T) + kron(S,I) )/h^2; % use spkron for Octave

 % form right hand side
 b = f(X(:),Y(:));

 % solve problem
 u = A\b;

 % compute max error
 err(i)=norm(u-utrue(X(:),Y(:)),'inf')

 % plot solution
 figure(1);
 hh=surf(X,Y,utrue(X,Y)); set(hh,'edgecolor','none');
 axis([0 1 0 1 -0.025 0.025]);
 xlabel('x'); ylabel('y');
 print('-depsc2',sprintf('prob1_n%d_true.eps',n));
 figure(2);
 hh=surf(X,Y,reshape(u,size(X))); set(hh,'edgecolor','none');
 axis([0 1 0 1 -0.025 0.025]);
 xlabel('x'); ylabel('y');
 print('-depsc2',sprintf('prob1_n%d_calc.eps',n));
end;

% log log plot of error
figure(3);
loglog(1./(ns+1),err);
xlabel('h'); ylabel('error');
print('-depsc2','prob1_err.eps');
```

HW 4 Problem 2 solutions

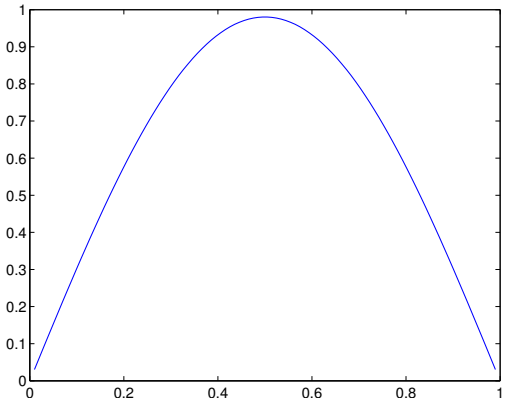
(a) (b)

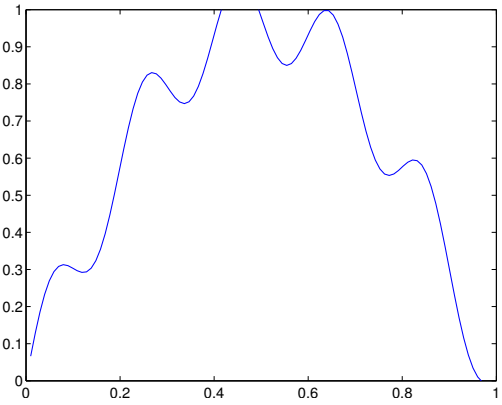
$t = 0$



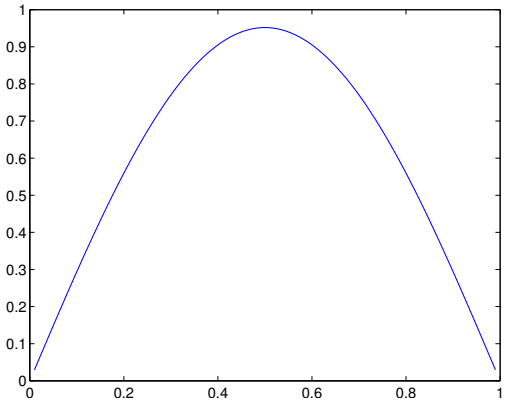


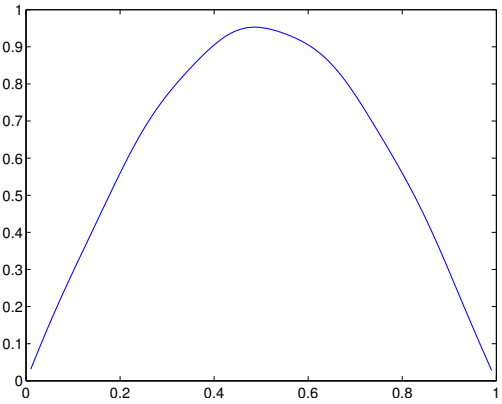
$t = k$





$t = 5k$





```
% Math 5620 HW 4 Problem 2
% Fernando Guevara Vasquez

% the initial condition

etas = { @(x) sin(pi*x)
         @(x) sin(pi*x) + sin(10*pi*x) };
for p = 1:2,

eta = etas{p};

% spatial discretization
n = 100; h = 1/(n+1);

e = ones(n,1);

% discrete Laplacian
A = (1/h^2)*spdiags([e -2*e e],-1:1,n,n);
I = speye(n,n);

% evaluate initial condition
u = eta(h*(1:n)');

% Crank-Nicholson
k = 0.001;
plot(h*(1:n),u); axis([0 1 0 1]);
print('-depsc2',sprintf('prob2_p%d_t%d.eps',p,0));
for i=1:5,
 u = (I - k*A/2)\( (I+k*A/2)*u );
 plot(h*(1:n),u); axis([0 1 0 1]);
 if (i==2 || i==5)
  print('-depsc2',sprintf('prob2_p%d_t%d.eps',p,i));
 end;
end;

end;
```