MATH 5620
HOMEWORK #2, DUE FRI FEB 11

(1) Show that if λ is an eigenvalue of A then $p(\lambda)$ is an eigenvalue of

$$p(A) = \sum_{i=0}^{n} a_i A^i.$$

(2) (K&C 4.6.2) Show that if A has this property (unit row diagonally dominant):

$$a_{ii} = 1 > \sum_{j=1,\ j\neq i} |a_{ij}|, \text{ for } 1 \leq i \leq n,$$

then Richardson iteration converges. **Hint:** Use the $\|\cdot\|_\infty$ induced matrix norm.

(3) (K&C 4.6.5,4.6.6) Let $S \in \mathbb{R}^{n\times n}$ be a non-singular matrix.
   (a) Let $\|\cdot\|$ be a norm in $\mathbb{R}^n$. Define $\|x\|' = \|Sx\|$. Show that $\|\cdot\|'$ is a norm.
   (b) Let $\|\cdot\|$ be an induced matrix norm. Define $\|A\|' = \|SAS^{-1}\|$. Show that $\|\cdot\|'$ is an induced matrix norm. **Hint:** Change variables in the sup.

(4) Please do parts (a) and (b) of the following problems.
- B&F 9^{th} ed: 7.3.6, 7.3.8 and 7.4.6.
- B&F 8^{th} ed: 7.3.6, 7.3.8 and 7.3.14.

Please include the first couple of iterates and also the last one. In addition create a table with the number of iterations needed to the reach the specified tolerance, similar to

	Jacobi	Gauss-Seidel	SOR $\omega = 1.2$
(a)	x	x	x
(b)	x	x	x

For this problem please stick with the algorithms of the class textbook. Do not use Matlab's backslash to solve the diagonal (or triangular) systems.

(5) (Trefethen and Bau, §40)

(a) Write the preconditioned conjugate gradient (CG) algorithm (see p58, 4^{th} set of class notes). Be sure to test your code on a small system. I suggest the following interface:

```
% Inputs
%   A          (nxn matrix) matrix in system to solve
%   x0         (nx1 vector) initial guess for solution
%   b          (nx1 vector) right hand side in system to solve
%   M          (nxn matrix) preconditioner
%   tol        tolerance to stop iterations
%   maxiter    maximum number of iterations
% Outputs
%   x          solution within tolerance to Ax=b
%   res        (vector of length number of iterations)
%              history of residuals
function [x,res] = mycg(A,x0,b,M,tol,maxiter)
```

(b) Let $A \in \mathbb{R}^{1000\times1000}$ be a symmetric matrix with $a_{ii} = \sqrt{i} + 0.5$ and $a_{ij} = 1$ when $|i - j| = 1$ or $|i - j| = 100$. Such a matrix can be generated in sparse form as follows:

```
D = ones(1000,5); D(:,3) = sqrt(1:1000)+0.5;
A = spdiags(D,[-100,-1,0,1,100],1000,1000);
```

Produce a `semilogy` plot with the number of iterations in the abscissa and the residual in the ordinate comparing the convergence history for the unpreconditioned CG and the preconditioned CG for solving the system $Ax = b$ with initial guess $x = 0$ and

$$b = (1, \ldots, 1)^T.$$

For the preconditioner use Jacobi, i.e. the diagonal part of A. **Note:** You do not need to write the un-preconditioned CG as it corresponds to setting $M = I$ (identity) in the preconditioned code.