MATH 5620 NUMERICAL ANALYSIS II
MIDTERM REVIEW SHEET

Chapter 6 – Direct methods for linear systems

- Know how to do Gaussian elimination on a simple matrix (say up to 4×4). Partial pivoting. I could give an example similar to the ones we saw in class where things break down for Gaussian elimination but where the problems go away with pivoting.
- You do not need to remember the algorithm for the LU factorization, but you should know what the factorization means, what happens in the symmetric positive definite case (Cholesky). How much does it cost to compute L and U? What is the work involved in the forward and backward substitution? Can you write the algorithm for forward and/or backward substitution in pseudocode?
- What is a symmetric positive definite matrix?

Chapter 7 – Iterative methods for linear systems

- Neumann series, condition for convergence, spectral radius, induced matrix norm. Iterative refinement.
- Jacobi, Gauss-Seidel, SOR. Advantages, disadvantages and pseudocode.
- Condition number error bound (7.19) and interpretation. We did not see Theorem 7.29 and it is not included in the exam.
- Conjugate gradient: do not memorize algorithm. You should be able to explain what are the advantages and disadvantages of this method with respect to direct methods. Why is CG a good method for sparse matrices? What is a preconditioner?

Chapter 5 – Initial Value Problems

- §5.11 Stiff differential equations. A-stability and how to determine if a linear multi-step method is A-stable. What is the region of absolute stability and what does it mean? (both are linked to how the method fares with the simple problem $y' = \lambda y; y(0) = 1$ with a given time step h).

Chapter 11 – Boundary Value Problems

- Theorem giving sufficient condition for the problem to have a unique solution.

- §11.1–11.2: Linear shooting method and Non-Linear Shooting method. You should know the basic ideas behind these two methods. Without knowing the formulas by heart you should be able to to linearly combine two solutions to a linear IVP to get the solution to a linear BVP. Also you should understand how non-linear shooting works (with secant and Newton's method)
- §11.3–11.4: Finite Difference Methods. For linear BVP know how to construct the linear system. For Non-Linear BVP: know how to setup the non-linear system (Jacobian etc. . .) and Newton's method.

Chapter 12 – Numerical solution to partial differential equations

Note: What we saw in class is significantly different from what is covered in the class textbook, so please refer to the class notes.

- **Elliptic problems:** (Laplace equation $\Delta u = f$) Five point stencil for Laplacian and how the discretization works. Know that the local truncation error is $\mathcal{O}(h^2)$ and how to derive it.
- **Parabolic problems:** (heat equation $u_t = \kappa \Delta u$).
  - Method of lines discretization. Euler and implict Trapezoidal rule (Crank-Nicholson).
  - Please do not remember the expressions for the eigenvalues of the discrete Laplacian. However you should be able to use these expressions to determine the region of stability of a discretization method: for example for Euler's method you should be able to get that $k/h^2 < 1/2$ from the expression of the eigenvalues for the discrete Laplacian.

Other considerations

- Understanding of the methods will be favored over simple memorization.
- You should be able to evaluate a method based on its practical merits (does it take too many function evaluations?. . .)
- Expect to write pseudo-code
- You are expected to know how to solve 2×2 systems (and simple 3×3 or triangular systems). Also you should know how to find the roots of a quadratic or a simple cubic or quartic polynomial.
- A calculator is not needed for the exam, and is not allowed.
- A one-sided, letter sized, handwritten cheat sheet is allowed.
- No books or other notes are allowed.