MATH 5620 CLASS PROJECT SUGGESTIONS
NUMERICAL ANALYSIS II

1. PROJECT GUIDELINES

- It is possible to do projects in pairs. Please inform me about this before you choose your project so that we can adjust the workload accordingly.
- Two persons (or groups) can do the same project
- A short talk (10-20min) where you will present your results to the class will be part of your project grade (20% talk, 80% write up). The time for your talk is flexible, just let me know well in advance if you will need more time.
- Please do **keep me informed on the progress** of your project.
- Please do let me know by email when you have chosen a project.
- You are **strongly encouraged** to come discuss the project with me, to make sure you are on the right track and that we agree on the outcome of the project.

2. TENTATIVE SCHEDULE

- Choose a project by Mon 03/29.
- Project reports are due the last day of classes Wed 04/28.
- Project presentations are on the last week of classes (04/26–04/28).

3. SUGGESTED CLASS PROJECTS

This is by no means an exclusive list. If you are interested in a particular application, we can figure out a numerical analysis project based on it. Don't be thrown off by the name of the method, some of them are very simple. If you need any of the referenced books I own them and would be happy to lend them to you. Also the referenced papers are from journals that our university subscribes to. Let me know if you have problems obtaining them.

1. A multigrid method for a 1D elliptic equation [3, 1].
2. A domain decomposition method for a 2D elliptic equation [2] (longer)
3. Model propagation of fire using the Eikonal equation [5] (longer, better adapted for two persons)
4. An example where the Runge-Kutta-Fehlberg method fails [6].
5. A simple mesh generator: [4].
6. Use a finite differences method for the viscous Burger's equation:

$$u_t + uu_x = \epsilon u_{xx}.$$

7. Compare the ADI, LOD and Crank-Nicholson methods for solving the heat equation in 2D on the unit square.
8. Implement the Chebyshev Spectral method for solving $u'' = f(x)$ [7, 3]
9. Two dimensional advection-diffusion equation with a constant velocity field.
10. Use the singular value decomposition to solve an ill-posed linear inverse problem (e.g. deblurring [8]).

4. Report and talk

What I expect is that you identify the problem (why is the method used?), tell me how the method deals with the problem and show a (simple) implementation of the method. You have to convince me in your report that you have understood the method. For the talk you would present the method to the class in a way that your class mates should be able to understand and show some numerical results.

References

[1] William L. Briggs, Van Emden Henson, and Steve F. McCormick. *A multigrid tutorial*. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, second edition, 2000.

[2] Tony F. Chan and Tarek P. Mathew. Domain decomposition algorithms. In *Acta numerica, 1994*, Acta Numer., pages 61–143. Cambridge Univ. Press, Cambridge, 1994.

[3] Randall J. LeVeque. *Finite difference methods for ordinary and partial differential equations*. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2007. Steady-state and time-dependent problems.

[4] Per-Olof Persson and Gilbert Strang. A simple mesh generator in Matlab. *SIAM Rev.*, 46(2):329–345 (electronic), 2004.

[5] J. A. Sethian. *Level set methods and fast marching methods*, volume 3 of *Cambridge Monographs on Applied and Computational Mathematics*. Cambridge University Press, Cambridge, second edition, 1999. Evolving interfaces in computational geometry, fluid mechanics, computer vision, and materials science.

[6] Joseph D. Skufca. Analysis still matters: a surprising instance of failure of Runge-Kutta-Felberg ODE solvers. *SIAM Rev.*, 46(4):729–737 (electronic), 2004.

[7] Lloyd N. Trefethen. *Spectral methods in MATLAB*, volume 10 of *Software, Environments, and Tools*. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2000.

[8] Curtis R. Vogel. *Computational methods for inverse problems*, volume 23 of *Frontiers in Applied Mathematics*. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2002. With a foreword by H. T. Banks.