MATH 5620 NUMERICAL ANALYSIS II
HOMEWORK 1, DUE FEBRUARY 1ST 2010

Problem 1 B&F 5.1.4 a,b.

Notes:

- Theorem 5.3 is a sufficient condition for a function satisfying a Lipschitz condition. It is not a necessary condition as there are functions that are not differentiable but that are Lipschitz continuous.
- Theorem 5.6 gives a sufficient condition for the well-posedness of an IVP. This condition is not necessary, so Theorem 5.6 does not rule out well-posedness if the Lipschitz condition is not satisfied.

Problem 2 (K&C 8.2.2) Consider the initial value problem:

$$\begin{cases} y' = \sqrt{y} \\ y(0) = 0. \end{cases}$$

(a) Verify that the function $y(t) = t^2/4$ is a solution to this IVP.
(b) Apply Euler's method to this IVP by hand (i.e. with $h > 0$ and $t_i = ih$ find y_i for $i = 0, 1, \ldots$).
(c) Explain why the numerical solution differs from the solution $t^2/4$. (no proof necessary)

Problem 3 In the following problems you need to compare several numerical methods for initial value problems on two problems. Compare the results to the actual values (the true solutions are given in B&F 5.2.3)

B&F 5.2.1 a,c (Euler's method)
B&F 5.3.1 a,c (Taylor's method of order 2)
B&F 5.4.1 a,c (Modified Euler's method)
B&F 5.4.5 a,c (Heun's method)
B&F 5.4.13 a,c (Runge-Kutta method of order four)

I suggest you use functions with prototype

```
function y = euler(a,b,n,y0,f)
```

Here `f` is a function handle for $f(t, y)$ and the output `y` would be a vector containing all time steps. For Taylor's method of order 2 you need partial derivative(s) of $f(t, y)$ that you can compute and give as extra parameter(s) (as function handles as well).

Problem 4 B&F 5.5.3 a,b (Runge-Kutta-Fehlberg method)