Problem 1 B&F 9.1.2, 9.1.4 (by hand please)
Problem 2 B&F 9.2.2 a,b and 9.2.8 a,b (power method)
Problem 3 B&F 9.2.6 a,c and 9.2.12 a,c (symmetric power method)
Problem 4 B&F 9.2.6 a,c and 9.2.12 a,c, but use Rayleigh Quotient Iteration (page 63 in the class notes) instead of the symmetric power method.
Problem 5 B&F 9.4.2 a,c and 9.4.4 a,c (QR Algorithm). Please do not implement Algorithm 9.6 in B&F. A simpler algorithm is outlined below.

Here are some implementation guidelines:

- Please display at least 7 digits of precision (or use format long)
- In Problems 2–5 I expect to see a few iterates of the method in question on the specified matrix and then the approximation to the eigenvalues (and eigenvectors in Problems 2–4). You can always check that your eigenvalues are correct using Matlab’s command eig (try [V,D]=eig(A)). Of course the eigenvectors you get maybe different from those given by eig because of normalization or signs.
- To check convergence in Problems 2–4 simply determine if two consecutive iterates (eigenvalues or eigenvectors) are within the desired tolerance (this is not an industrial strength convergence criterion!)
- In Problem 5: Please use directly Matlab’s qr function. The algorithm in p73 (notes) needs some kind of deflation technique to work well. Deflation simply means that when the desired accuracy of an eigenvalue is met the procedure continues with the next eigenvalue. To detect convergence of the entry $T_{m,m}$ to an eigenvalue of $T$ its enough to require that $|T_{m,m} - T_{m-1,m-1}| < \text{TOL}$. If $T_{m,m}$ is within the desired tolerance, we can set $m = m - 1$ and continue with $T(1 : m, 1 : m)$. See next page for a suggested implementation.
Given an \( n \times n \) tridiagonal matrix \( T \) and a prescribed tolerance \( \epsilon \), here is a practical shifted QR algorithm:

\[
m = n
\]

for \( k = 0, 1, \ldots \)

\[
\text{Determine shift } \mu \quad (* \text{Wilkinson shift recommended} *)
\]

\[
QR = T(1:m, 1:m) - \mu I \quad (* \text{QR factorization} *)
\]

\[
T(1:m, 1:m) = RQ + \mu I
\]

(* check convergence of \( T(m, m) \) to an eigenvalue of \( T \) *)

if \( |A(m, m - 1)| < \epsilon \) then

if \( m > 2 \) then

(* go to next eigenvalue *)

\( m = m - 1 \)

else

(* all eigenvalues have converged to within prescribed tolerance *)

print 'done in' \( k \) 'iterations'

stop

end if

end if

end for

• For determining the shift \( \mu \) please use the Wilkinson shift (the one given in the notes can have problems in pathological cases):

\[
d = \frac{1}{2}(A(m - 1, m - 1) - A(m, m))
\]

if \( d > 0 \)

\[
\mu = A(m, m) + d - \sqrt{d^2 + A(m, m - 1)^2}
\]

else

\[
\mu = A(m, m) + d + \sqrt{d^2 + A(m, m - 1)^2}
\]

end if