MATH 5620 NUMERICAL ANALYSIS II
MIDTERM REVIEW SHEET

1. Chapter 5 – Initial Value Problems

- §5.1 Lipschitz condition, well-posedness, existence and uniqueness theorem for first order IVP.
- §5.2-5.3: Taylor series method, know how to derive them. Notion of order, local truncation error and global truncation error (in general). How to find local truncation error for Taylor series method.
- §5.4: Runge-Kutta methods. Advantages over Taylor series methods. Know general form of second order Runge-Kutta method, and why the parameters are related (match Taylor series). You don't need to know all the RK2 methods by their names. Why are RK methods of order higher than 4 less attractive than RK4?
- §5.5: Runge-Kutta-Fehlberg method. You have to know the basic idea (using embedded methods). Please do not learn the method by heart or its coefficients. Need to understand the derivation of update for h.
- §5.6 Multistep methods. Understand the relationship with integration formulas. Use the undetermined coefficient method. Difference between Adams-Bashforth and Adams-Moulton. What is a predictor-corrector method?
- Analysis of linear multi-step methods. Know the linear functional L and how it can be used to get the order or the local truncation error of a method. You do not need to know the formulas for the d_j exactly, but you should know where they come from.
- §5.7 Variable step-size multistep methods. You need to know the big idea to obtain adaptivity. How does this method compare to Runge-Kutta-Fehlberg from the practical point of view? As for RKF you may be asked to estimate the size h to get a desired precision.
- §5.8 Extrapolation methods: **not included**.
- §5.9 Higher order equations and systems. Know how to transform an algorithm for the scalar case to system case. Know how to transform a higher order equation into a system.
- §5.10 Stability, only for *linear* multistep methods. Difference between implicit/explicit. Notion of convergent method. Consistency and stability notions and how they relate to convergence. Know how to determine if a given linear multistep method is consistent, stable or convergent. (you don't need to know the proof that convergence implies stability and consistency).

- Difference equations. Know how to solve a difference equation when roots of characteristic polynomial are simple (i.e. first theorem on p33 in the notes)
- §5.11 Stiff differential equations. A-stability and how to determine if a linear multi-step method is A-stable. What is the region of absolute stability and what does it mean? (both are linked to how the method fares with the simple problem $y' = \lambda y; y(0) = 1$ with a given time step h).

2. Chapter 11 – Boundary Value Problems

- Theorem giving sufficient condition for the problem to have a unique solution.
- §11.1–11.2: Linear shooting method and Non-Linear Shooting method. You should know the basic ideas behind these two methods. Without knowing the formulas by heart you should be able to to linearly combine two solutions to a linear IVP to get the solution to a linear BVP. Also you should understand how non-linear shooting works (with Newton's method)
- §11.3–11.4: Finite Difference Methods. For linear BVP know how to construct the linear system. For Non-Linear BVP: know how to setup the non-linear system (Jacobian etc...) and Newton's method.

3. Other considerations

- Understanding of the methods will be favored over simple memorization.
- About 70% of the exam will be on IVP and 30% on BVP.
- You should be able to evaluate a method based on its practical merits (does it take too many function evaluations?...)
- Expect to write pseudo-code
- You are expected to know how to solve 2×2 systems (and simple 3×3 or triangular systems). Also you should know how to find the roots of a quadratic or a simple cubic or quartic polynomial.
- A calculator is not needed for the exam, but it is allowed.
- A one page cheat sheet is allowed.