Math 5610/6860
Homework #5, due Tue Nov 8

Note: Please see the sample code for Prob 1 in class website.

1. B&F 3.5.4(d) and 3.5.6(d). Produce a plot of the points and their natural cubic spline interpolant (as in the sample code). The easiest way is to proceed as in Example 2 p150, i.e. form the system of Theorem 3.11, solve it for the coefficients of the spline in each subinterval and then evaluate. You are not allowed to use Matlab's `spline` or `interp1` for this problem.
2. Assume every time we compute a function $f(x)$ we compute in fact $f_\epsilon(x)$, such that $|f(x) - f_\epsilon(x)| \le \epsilon$. This systematic error could come from measurement errors or simply from using floating point to represent $f(x)$. The error made with the forward difference approximation to the derivative using f_ϵ instead of f is
$$\left| \frac{f_\epsilon(x+h) - f_\epsilon(x)}{h} - f'(x) \right| \le 2\frac{\epsilon}{h} + \frac{h}{2}M,$$
for some ξ between x and $x+h$ and where M is such that $|f''(\xi)| \le M$ for $\xi \in [x, x+h]$.
   (a) What is the step size h_* that minimizes this upper bound?
   (b) Let $f(x) = e^x$. Plot in log log scale (`loglog` in Matlab) the approximation error for $f'(1)$ using the forward difference formula for `h = 10.^(0:-1:-20)`
   (c) Explain your results. Is the best step size h in your graph consistent with the h_* you derived?
3. (K&C 7.2.12) Derive a formula for approximating
$$\int_1^3 f(x)dx$$
in terms of $f(0)$, $f(2)$ and $f(4)$. Your formula should be exact for all polynomials of degree ≤ 2. **Note:** The node $x = 4$ is indeed outside of the integration interval. Use the undetermined coefficients method.
4. B&F 4.3.16. To find the degree of precision you need to find the largest k such that the formula is exact for the monomials $x^0, x^1, \ldots, x^k$.
5. B&F 4.4.8 a,b,c. (Comparison of composite trapezoidal, midpoint and Simpson's quadratures)
6. B&F 4.7: 1 through 4 for functions (a) and (b). (Gaussian quadrature)