MATH 5610/6860
HOMEWORK #5, DUE THU OCT 29

**Note:** choose any 6 (7 for Math 6860 students) problems out of the 8 problems below for full credit. Any additional problems will be counted as extra credit.

1. B&F 3.4.4(d) and 3.4.6(d). Produce a plot of the points and their natural cubic spline interpolant. The easiest way is to proceed as in Example 1 p143, i.e. form the system in p142, solve it for the coefficients of the spline in each subinterval and then evaluate.

2. Assume every time we compute a function \( f(x) \) we compute in fact \( f_\epsilon(x) \), such that \( |f(x) - f_\epsilon(x)| \leq \epsilon \). This systematic error could come from measurement errors or simply from using floating point to represent \( f(x) \). We showed in class that the error made with the forward difference approximation to the derivative using \( f_\epsilon \) instead of \( f \) is

\[
\left| \frac{f_\epsilon(x+h) - f_\epsilon(x)}{h} - f'(x) \right| \leq \frac{2\epsilon}{h} + \frac{h}{2} |f''(\xi)|,
\]

for some \( \xi \) between \( x \) and \( x+h \).

(a) Assuming there is some \( M > 0 \) such that \( |f''(\xi)| \leq M \) for all \( \xi \), what is the best value \( h^* \) for the step size?

(b) Let \( f(x) = e^x \). Plot in log log scale (\texttt{loglog} in Matlab) the approximation error for \( f'(1) \) using the forward difference formula for \( h = 10.\^\texttt{(0:-1:-20)} \).

(c) Explain your results. Is the best step size \( h \) in your graph consistent with the \( h^* \) you derived?

3. (K&C 7.2.12) Derive a formula for approximating

\[
\int_1^3 f(x)dx
\]

in terms of \( f(0), f(2) \) and \( f(4) \). Your formula should be exact for all polynomials of degree \( \leq 2 \). Note: Since the node \( x = 4 \) is outside of the bounds of the integral, use the undetermined coefficient method.
4. (K&C 7.2.23) Prove that if the formula
\[
\int_{-1}^{1} f(x) \, dx \approx \sum_{i=0}^{n} A_i f(x_i), \quad (n \text{ is even}),
\]
is exact for all polynomials of degree \( n \), and if the nodes are symmetrically placed about the origin then the formula is exact for all polynomials of degree \( \leq n + 1 \).

5. B&F 4.3.16
6. B&F 4.4.8 a,b,c
7. B&F 4.7: 1 through 4 for functions (a) and (b).
8. B&F 4.7.6: Use the undetermined coefficients method to build a system of linear equations with unknowns \( a, b, c, d, e \). You may solve this linear system using Matlab or a calculator. (See \texttt{help slash} for help on the Matlab “backslash” operator). Note there is a small typo in the book: the beginning of the sentence should read “Determine constants \( a, b, c, d \) and \( e \)”