MATH 5610/6860
HOMEWORK #5, DUE THU OCT 29

Note: choose any 6 (7 for Math 6860 students) problems out of the 8 problems below for full credit. Any additional problems will be counted as extra credit.

1. B&F 3.4.4(d) and 3.4.6(d). Produce a plot of the points and their natural cubic spline interpolant. The easiest way is to proceed as in Example 1 p143, i.e. form the system in p142, solve it for the coefficients of the spline in each subinterval and then evaluate.
2. Assume every time we compute a function $f(x)$ we compute in fact $f_\epsilon(x)$, such that $|f(x) - f_\epsilon(x)| \leq \epsilon$. This systematic error could come from measurement errors or simply from using floating point to represent $f(x)$. We showed in class that the error made with the forward difference approximation to the derivative using f_ϵ instead of f is
$$\left| \frac{f_\epsilon(x+h) - f_\epsilon(x)}{h} - f'(x) \right| \leq 2\frac{\epsilon}{h} + \frac{h}{2}|f''(\xi)|,$$
for some ξ between x and $x+h$.
   (a) Assuming there is some $M > 0$ such that $|f''(\xi)| \leq M$ for all ξ, what is the best value h_* for the step size?
   (b) Let $f(x) = e^x$. Plot in log log scale (`loglog` in Matlab) the approximation error for $f'(1)$ using the forward difference formula for `h = 10.^(0:-1:-20)`
   (c) Explain your results. Is the best step size h in your graph consistent with the h_* you derived?
3. (K&C 7.2.12) Derive a formula for approximating
$$\int_1^3 f(x)dx$$
in terms of $f(0)$, $f(2)$ and $f(4)$. Your formula should be exact for all polynomials of degree ≤ 2. Note: Since the node $x = 4$ is outside of the bounds of the integral, use the undetermined coefficient method.

4. (K&C 7.2.23) Prove that if the formula
$$\int_{-1}^{1} f(x)dx \approx \sum_{i=0}^{n} A_i f(x_i), \ (n \text{ is even}),$$
is exact for all polynomials of degree n, and if the nodes are symmetrically placed about the origin then the formula is exact for all polynomials of degree $\leq n+1$.
5. B&F 4.3.16
6. B&F 4.4.8 a,b,c
7. B&F 4.7: 1 through 4 for functions (a) and (b).
8. B&F 4.7.6: Use the undetermined coefficients method to build a system of linear equations with unknowns a, b, c, d, e. You may solve this linear system using Matlab or a calculator. (See `help slash` for help on the Matlab "backslash" operator). Note there is a small typo in the book: the beginning of the sentence should read "Determine constants a, b, c, d and e..."