MATH 5610/6860
HOMEWORK #4, DUE MON OCT 5

Notes: Problems denoted by [**EC**] are extra-credit for Math 5610 students, but **required** for Math 6860 students.

1. B&F 2.6.4 a,h (Müller's method)
2. B&F 3.1.2 a,b and 3.1.4 a,b (interpolation by hand using Lagrange interpolation polynomials)
3. K&C 6.2.23. The polynomial

   $$p(x) = 2 - (x+1) + x(x+1) - 2x(x+1)(x-1)$$

   interpolates the first four points in the table

x	-1	0	1	2	3
y	2	1	2	-7	10

   By adding one additional term to p, find a polynomial that interpolates the whole table.
4. B&F 3.1.6 a,b, additionally implement the divided differences algorithm 3.2 to obtain the same results. Then compare the actual interpolation error to a bound derived from Theorem 3.3 (i.e. B&F 3.1.10 a,b)
5. K&C 6.2.18 Prove that if f is a polynomial, then the divided differences $f[x_0, x_1, \ldots, x_n]$ is a polynomial in the variables $x_0, x_1, \ldots, x_n$. (i.e. if we freeze all the x_j for $j \neq i$, then $q(x_i) \equiv f[x_0, x_1, \ldots, x_n]$ is a polynomial in x_i). You may prove this by an induction argument:
   i. Prove that $f[x_0]$ is a polynomial in x_0
   ii. Prove that $f[x_0, x_1]$ is a polynomial in x_0 and x_1 (not needed for induction argument but it helps to see what is going on)
   iii. Assuming n-th order divided differences are polynomials of their variables, prove the statement for $(n+1)$-th order divided differences (Hint: use B&F eq. (3.9))
6. [**EC**] K&C 6.2.7 and 6.2.13. The divided difference $f[x_0, x_1]$ is analogous to the first derivative (see B&F, Theorem 3.6).
   (a) Show that it has a property analogous to $(fg)' = f'g + g'f$ (see the general form below for a hint on what to prove)
   (b) Prove the **Leibniz formula**

   $$(fg)[x_0, x_1, \ldots, x_n] = \sum_{k=0}^{n} f[x_0, x_1, \ldots, x_k] g[x_k, x_{k+1}, \ldots, x_n].$$