Notes: Problems denoted by [EC] are extra-credit for Math 5610 students, but **required** for Math 6860 students.

1. B&F 2.6.4 a,h (Müller’s method)
2. B&F 3.1.2 a,b and 3.1.4 a,b (interpolation by hand using Lagrange interpolation polynomials)
3. K&C 6.2.23. The polynomial
   \[ p(x) = 2 - (x + 1) + x(x + 1) - 2x(x + 1)(x - 1) \]
   interpolates the first four points in the table
   \[
   \begin{array}{c|cccc}
   x & -1 & 0 & 1 & 2 \\
   \hline
   y & 2 & 1 & 2 & -7 \\
   \end{array}
   \]
   By adding one additional term to \( p \), find a polynomial that interpolates the whole table.

4. B&F 3.1.6 a,b, additionally implement the divided differences algorithm 3.2 to obtain the same results. Then compare the actual interpolation error to a bound derived from Theorem 3.3 (i.e. B&F 3.1.10 a,b)

5. K&C 6.2.18 Prove that if \( f \) is a polynomial, then the divided differences \( f[x_0, x_1, \ldots, x_n] \) is a polynomial in the variables \( x_0, x_1, \ldots, x_n \). (i.e. if we freeze all the \( x_j \) for \( j \neq i \), then \( q(x_i) \equiv f[x_0, x_1, \ldots, x_n] \) is a polynomial in \( x_i \)). You may prove this by an induction argument:
   i. Prove that \( f[x_0] \) is a polynomial in \( x_0 \)
   ii. Prove that \( f[x_0, x_1] \) is a polynomial in \( x_0 \) and \( x_1 \) (not needed for induction argument but it helps to see what is going on)
   iii. Assuming \( n \)-th order divided differences are polynomials of their variables, prove the statement for \( (n + 1) \)-th order divided differences (Hint: use B&F eq. (3.9))

6. [EC] K&C 6.2.7 and 6.2.13. The divided difference \( f[x_0, x_1] \) is analogous to the first derivative (see B&F, Theorem 3.6).
   (a) Show that it has a property analogous to \( (fg)' = f'g + gf' \)
      (see the general form below for a hint on what to prove)
   (b) Prove the **Leibniz formula**
      \[
      (fg)[x_0, x_1, \ldots, x_n] = \sum_{k=0}^{n} f[x_0, x_1, \ldots, x_k]g[x_k, x_{k+1}, \ldots, x_n].
      \]