MATH 5610/6860
HOMEWORK #2, DUE THU SEP 17

Notes: Problems marked with "[**EC**]" are extra credit for Math 5610 students but **required** for Math 6860 students.

1. B&F 2.1.6 c,d (Bisection method in Matlab)
2. B&F 2.2.19. Additionally show that the iteration can be obtained by applying Newton's method to a certain function.
3. B&F 2.3.6 a,b and 2.3.8 a,b (Newton's method and Secant method)
4. K&C 3.2.16 Prove Newton's iteration will diverge for $f(x) = x^2 + 1$ no matter what (real) starting point is selected. (Hint: assume for contradiction that Newton's iteration has a limit, what relation should the limit satisfy?)
5. K&C 3.4.12 Let p be a positive number. What is the value of the following expression?
$$x = \sqrt{p + \sqrt{p + \sqrt{p + \cdots}}}$$
Note that this can be interpreted as meaning $x = \lim_{n\to\infty} x_n$, where $x_1 = \sqrt{p}$, $x_2 = \sqrt{p + \sqrt{p}}$, etc...
(Hint: observe that $x_{n+1} = \sqrt{p + x_n}$.)
6. K&C 3.4.18 Prove that if F' is continuous and if $|F'(x)| < 1$ on the interval $[a, b]$, then F is a contraction on $[a, b]$. Show that this is not necessarily true for an open interval.
7. K&C 3.4.25 Prove that the function F defined by $F(x) = 4x(1 - x)$ maps the interval $[0, 1]$ into itself and is not a contraction. Prove that it has a fixed point. Why does this not contradict the Contractive Mapping Theorem?
8. [**EC**] B&F 2.4.12 (Proof of theorem 2.11)