MATH 5610/6860
HOMEWORK #2, DUE THU SEP 17

Notes: Problems marked with “[EC]” are extra credit for Math 5610 students but required for Math 6860 students.

1. B&F 2.1.6 c,d (Bisection method in Matlab)
2. B&F 2.2.19. Additionally show that the iteration can be obtained by applying Newton’s method to a certain function.
3. B&F 2.3.6 a,b and 2.3.8 a,b (Newton’s method and Secant method)
4. K&C 3.2.16 Prove Newton’s iteration will diverge for \( f(x) = x^2 + 1 \) no matter what (real) starting point is selected. (Hint: assume for contradiction that Newton’s iteration has a limit, what relation should the limit satisfy?)
5. K&C 3.4.12 Let \( p \) be a positive number. What is the value of the following expression?

\[
x = \sqrt{p + \sqrt{p + \sqrt{p + \cdots}}}
\]

Note that this can be interpreted as meaning \( x = \lim_{n \to \infty} x_n \), where \( x_1 = \sqrt{p} \), \( x_2 = \sqrt{p + \sqrt{p}} \), etc...

(Hint: observe that \( x_{n+1} = \sqrt{p + x_n} \).)

6. K&C 3.4.18 Prove that if \( F' \) is continuous and if \( |F'(x)| < 1 \) on the interval \([a, b]\), then \( F \) is a contraction on \([a, b]\). Show that this is not necessarily true for an open interval.

7. K&C 3.4.25 Prove that the function \( F \) defined by \( F(x) = 4x(1-x) \) maps the interval \([0, 1]\) into itself and is not a contraction. Prove that it has a fixed point. Why does this not contradict the Contractive Mapping Theorem?

8. [EC] B&F 2.4.12 (Proof of theorem 2.11)