MATH 5610/6860
HOMEWORK #1, DUE THU SEP 10

Notes:

- Problems marked with "**[EC]**" are extra credit for Math 5610 students but **required** for Math 6860 students.
- The homework policy is in the class website (Aug 31 entry).
- Please read the guidelines for numerical experiments and submitting your homework electronically sections in the class website.
- Common abbreviations for problem sources are: B&F (Burden and Faires, the class textbook) and K&C (Kincaid and Cheney).

1. B&F 1.1.1 a,b
2. B&F 1.1.3 a,b
3. B&F 1.1.7
4. B&F 1.2.3 a
5. B&F 1.3.3
6. K&C 1.2.7: Choose the correct assertions (in each, $n \to \infty$)
   a. $(n+1)/n^2 = o(1/n)$
   b. $(n+1)/\sqrt{n} = o(1)$
   c. $1/\ln n = \mathcal{O}(1/n)$
   d. $1/(n \ln n) = o(1/n)$
   e. $e^n/n^5 = \mathcal{O}(1/n)$
7. K&C 1.2.8: The expressions e^h, $(1-h^4)^{-1}$, $\cos(h)$, and $1+\sin(h^3)$ all have the same limit as $h \to 0$. Express each in the following form with the best integer values of α and β.
$$f(h) = c + \mathcal{O}(h^\alpha) = c + o(h^\beta)$$
8. K&C 2.1.10: Let $x = 2^3 + 2^{-19} + 2^{-22}$. Find the machine numbers (in IEEE single precision) that are just to the left and to the right of x. Determine $\mathrm{fl}(x)$ (rounding to the nearest machine number), the absolute error $|\mathrm{fl}(x) - x|$, and the relative error $|\mathrm{fl}(x) - x|/|x|$. Verify that the relative error in this case does not exceed $\epsilon/2 = 2^{-24}$.
9. Verify numerically that $\sin((2n-1)/n^3) = \mathcal{O}(1/n^2)$.
10. **[EC]** K&C 1.2.35: Show that $x_n = o(\alpha_n) \Rightarrow x_n = \mathcal{O}(\alpha_n)$. Show that the converse is not true.