

MATH 5610/6860
FINAL EXAM PRACTICE PROBLEMS #1

1. Find the cubic spline $S(x)$ for $x \in [0, 1]$ satisfying the conditions $S(0) = 0$, $S(1) = 1$, $S'(0) = S'(1) = 0$.
2. (K&C 7.1.15) Derive a numerical differentiation formula of order $\mathcal{O}(h^4)$ by applying Richardson's extrapolation to

$$f'(x) = \frac{1}{2h}[f(x+h) - f(x-h)] - \frac{h^2}{6}f'''(x) - \frac{h^4}{120}f^{(5)}(x) - \dots$$

What is the error in terms of h^4 ?

3. (K&C 7.2.10) Use the Lagrange interpolation polynomial to derive a quadrature formula of the form

$$\int_0^1 f(x)dx \approx Af(1/3) + Bf(2/3).$$

Transform this formula to one for integration over $[a, b]$.

4. (K&C 7.5.2) The trapezoid rule can be written in the form

$$I \equiv \int_u^v f(x)dx = T(u, v) - \frac{1}{2}(v-u)^3 f''(\xi).$$

- (a) Let $w = (u+v)/2$. Find the constant C in the error term below

$$I = T(u, w) + T(w, v) + C(v-u)^3 f''(\tilde{\xi}).$$

- (b) Assuming $f''(\tilde{\xi}) \approx f''(\xi)$ find an approximation to the *error term* in the equation from (a) in terms of $T(u, w)$, $T(w, v)$ and $T(u, v)$.