

Math 3150-4 – Midterm 2 – review sheet

- §3.4 (D'Alembert's method) You should know the form of D'Alembert's solution to the 1D Wave Equation in the general case (both initial condition and initial velocity are given). You also should be able to verify that solutions to the wave equation given by separation of variables can be written in D'Alembert's form (see Example 3.4.1). Characteristics and Interval of dependence are not included.
- §3.5 (1D Heat equation) You should know how to solve the 1D Heat equation with homogeneous Dirichlet boundary conditions with separation of variables. When the boundary conditions are Dirichlet non-homogeneous, the solution can be found by first finding the steady state heat distribution and subtracting it from the original problem to get a homogeneous one. Can you compare the rate of decay of two temperature distributions (say when they are made of pure modes, which decays faster, higher or lower frequencies?)
- §3.6 (1D Heat equation with other boundary conditions) You should be familiar with the case of homogeneous Neumann boundary conditions at both ends (Example 3.6.1). The key is to use cosine series instead of sine series, and the reason is that cosines satisfy the boundary conditions and sines do not! The case of one radiating end (Example 3.6.2) will not be included in the exam. However you are expected to know how to do the following mixed boundary cases for a bar $[0, L]$
 - Insulated at $x = 0$ and ice bath at $x = L$
 - Ice bath at $x = 0$ and insulated at $x = L$ (see Problem 3.6.5)
- §3.7 (2D WEQ and HEQ in rectangular domains with homogeneous Dirichlet boundary conditions) You should know how to do example 3.7.1 (and also the non-zero velocity case too). Since the double sine series coefficients calculation can be lengthy, it is very likely that either the coefficients will be given or the initial position or velocity of the membrane are given already in double sine series form (as in problem 3.7.1 for example). The solution to the 2D heat equation on a rectangular domain uses exactly the same double sine series tool and you should also be familiar with it (see example 3.7.2).
- §3.8 (2D Laplace equation in rectangular coordinates) Here the trick is to reduce the problem with boundary conditions on all four sides of the rectangle to 4 problems, each with a non-zero boundary condition on one side only. You only need to know how to do one of these problems (say the one in Example 3.8.1). The other 3 problems can be obtained by symmetries.
- §4.1: Here you should know what are polar coordinates and what is the Laplacian in polar coordinates. The derivation is not included in the exam

(it is too long). You may be asked to compute $\partial r/\partial x$, $\partial r/\partial y$, $\partial\theta/\partial x$, $\partial\theta/\partial y$ using the chain rule or a problem similar to problem 4.1.1 (i.e. check whether an expression which is much simpler in polar coordinates is a solution to Laplace equation).

- §4.2 (Vibrations of circular membrane in the case where the initial conditions are independent of θ) You should know Theorem 4.2.1 and example 4.2.1 (without the numerical calculations of the actual coefficients of course). Since the calculations of the coefficients in a Bessel series expansion are hard to do by hand, you will likely be given the initial condition and/or initial velocity in Bessel series expansion form already or be provided with Bessel function identities such as those used in Example 4.2.2. You may also be asked to consider the heat equation on a disk with radial symmetry (i.e. with initial condition independent of θ), see Problem 4.3.10.