

3.3.2 (a) $f(x) = \sin \pi x \cos \pi x$, $g(x) = 0$, $c = \frac{1}{\pi}$, $L = 1$

Here $g(x) = 0 \Rightarrow b_n^* = 0$ and sol is:

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin n\pi x \cos nt$$

where $b_n = 2 \int_0^1 \sin n\pi x \sin \pi x \cos \pi x dx$
 $\quad \quad \quad = \int_0^1 \sin \pi x \sin (2\pi x) dx$ ✓ dble angle formula
 $\quad \quad \quad = \begin{cases} \frac{1}{2} & \text{if } n=2 \\ 0 & \text{otherwise.} \end{cases}$

$$\Rightarrow \boxed{u(x,t) = \frac{1}{2} \sin (2\pi x) \cos 2t}$$

(b) See attached code & plot.

3.3.3 $f(x) = \sin(\pi x) + 3 \sin(2\pi x) - \sin 5\pi x$
 $g(x) = 0$
 $c = 1$, $L = 1$

$$\Rightarrow u(x,t) = \sum_{n=1}^{\infty} b_n \sin n\pi x \cos n\pi t$$

where $b_n = 2 \int_0^1 \sin(n\pi x) f(x) dx = \begin{cases} 1 & \text{if } n=1 \\ 3 & \text{if } n=2 \\ -1 & \text{if } n=5 \\ 0 & \text{otherwise} \end{cases}$

$$\Rightarrow \boxed{u(x,t) = \sin(\pi x) \cos \pi t + 3 \sin(2\pi x) \cos(2\pi t) - \sin(5\pi x) \cos(5\pi t)}$$

(b) See attached code & plot

3.3.4 $f(x) = \sin \pi x + \frac{1}{2} \sin 3\pi x + 3 \sin 7\pi x$

$$g(x) = \sin(2\pi x)$$

$$c = 1, L = 1$$

Sol to IDWFA is:

$$u(x, t) = \sum_{n=1}^{\infty} \sin n\pi x (b_n \cos(n\pi ct) + b_n^* \sin(n\pi ct))$$

where $b_n = 2 \int_0^1 f(x) \sin(n\pi x) dx =$ sine series coeff of $f(x)$.

Here sine series of f is itself:

$$b_1 = 1$$

$$b_3 = \frac{1}{2}$$

$$b_7 = 3$$

$$b_n = 0 \text{ for } n \neq 1, 3, 7.$$

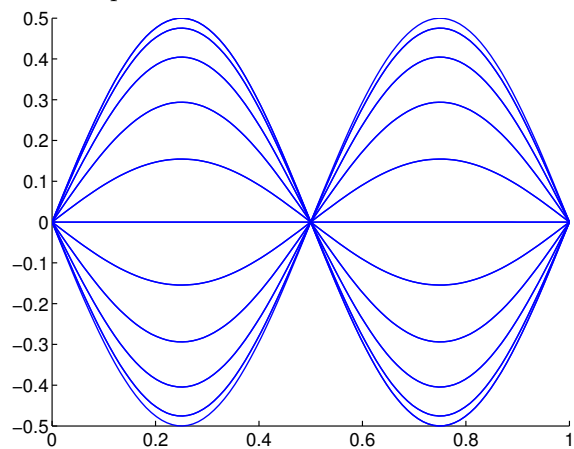
We also have $b_n^* = \frac{2}{n\pi} \int_0^1 g(x) \sin n\pi x dx = \frac{1}{n\pi} \times$ sine series coeff of $g(x)$

$$\Rightarrow b_n^* = \begin{cases} \frac{1}{2\pi} & \text{if } n=1 \\ 0 & \text{if } n \neq 1 \end{cases}$$

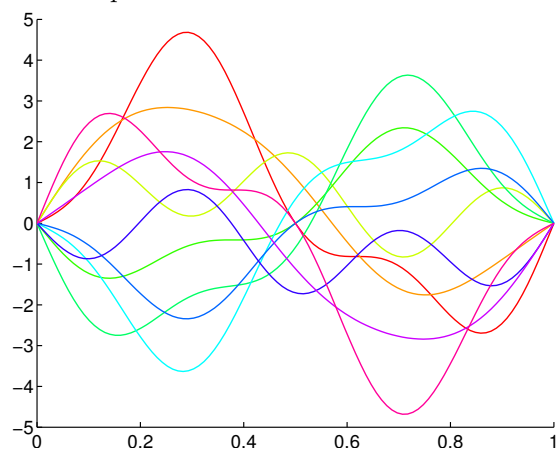
Thus:

$$u(x, t) = \sin \pi x \cos \pi t + \frac{1}{2} \sin 3\pi x \cos 3\pi t + 3 \sin 7\pi x \cos 7\pi t + \frac{1}{2\pi} \sin(2\pi x) \sin(2\pi t)$$

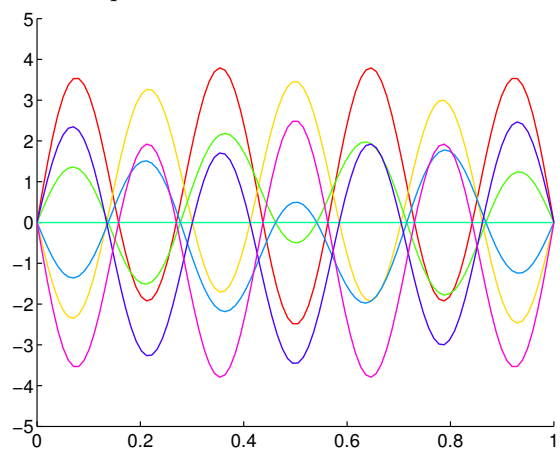
Plot for problem 3.3.2



Plot for problem 3.3.3



Plot for problem 3.3.4



```

1 % Math 3150 Fall 2012
2 %
3 % Problem 3.3.2
4
5 thickLines(3); % remove if it is not in your
   system
6
7 % space variable
8 x = linspace(0,1);
9
10 % time variable will take values in this array
11 ts=linspace(0,pi,21);
12 figure(1); clf;
13 hold on;
14 for it=1:length(ts), % for every time
15     t = ts(it);
16     % we don't need to loop over the number of terms
17     % the sum has only ONE term
18     s = sin(2*pi*x)*cos(2*t)/2;
19     plot(x,s); % plot the partial sum
20     % adjust axis so that all plots are on the same
       scale
21     axis([0 1 -0.5 0.5]);
22 end;
23 hold off;
24 filename='p3_3_2.eps';
25 print('-depsc2',filename);
26 system(['epstopdf ',filename]);

```

```

1 % Math 3150 Fall 2012
2 %
3 % Problem 3.3.3
4
5 thickLines(3); % remove if is not in your system
6
7 % space variable
8 x = linspace(0,1);
9
10
11 % time variable will take values in this array
12 nts = 10; % number of times at which to plot
       string
13 ts=linspace(0,1,nts);
14 cols = hsv(nts); % colors with which to plot
       string
15 figure(1); clf;
16 hold on;
17 for it=1:length(ts), % for every time
18     t = ts(it);
19     % we don't need to loop over the number of terms
20     % the sum has only a few terms
21     s = sin(pi*x)*cos(pi*t) + 3*sin(2*pi*x)*cos(2*pi
       *t) - sin(5*pi*x)*cos(5*pi*t);
22     h=plot(x,s); % plot the partial sum
23     set(h,'color',cols(it,:));
24     % adjust axis so that all plots are on the same
       scale
25     axis([0 1 -5 5]);
26     %pause;
27 end;
28 hold off;
29 filename='p3_3_3.eps';
30 print('-depsc2',filename);
31 system(['epstopdf ',filename]);

```

```

1  % Math 3150 Fall 2012
2  %
3  % Problem 3.3.4
4
5  thickLines(3); % remove if is not in your system
6
7  % space variable
8  x = linspace(0,1);
9
10
11 % time variable will take values in this array
12 nts = 7; % number of times at which to plot
    string
13 ts=linspace(0,1,nts);
14 cols = hsv(nts); % colors with which to plot
    string
15 figure(1); clf;
16 hold on;
17 for it=1:length(ts), % for every time
18     t = ts(it);
19     % we don't need to loop over the number of terms
20     % the sum has only a few terms
21     s = sin(pi*x)*cos(pi*t) + (1/2)*sin(3*pi*x)*cos
        (3*pi*t) + 3*sin(7*pi*x)*cos(7*pi*t) + (1/2/
        pi)*sin(2*pi*x)*sin(2*pi*t);
22     h=plot(x,s); % plot the partial sum
23     set(h,'color',cols(it,:));
24     % adjust axis so that all plots are on the same
        scale
25     axis([0 1 -5 5]);
26     %pause;
27 end;
28 hold off;
29 filename='p3-3.4.eps';
30 print('—depsc2',filename);
31 system(['epstopdf ',filename]);

```