MATH 3150 HW4 solutions

2.3.2 (a) $f(x) = x$ if $-p < x < p$, $2p$ periodic, odd function with discontinuities at $(2k+1)p$, $k \in \mathbb{Z}$.

(b) From problem 2.2.13: $g(x) = x$ if $-\pi < x < \pi$, 2π-per has Fourier series:

$$g(x) = 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx$$

$$\boxed{f(x) = \frac{p}{\pi} g\left(\frac{\pi}{p} x\right) = \frac{2p}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin \frac{n\pi x}{p}}$$

2.3.7 (a) $f(x) = \begin{cases} -\frac{2}{p}(x - p/2) & \text{if } 0 < x < p \\ -\frac{2}{p}(x + p/2) & \text{if } -p < x < 0 \end{cases}$

$2p$-periodic ODD function disc. at $x = kp$, $k \in \mathbb{Z}$.

(b) The sine series coeff of $f(x)$ are:

$$b_n = \frac{2}{p} \int_0^p \left(\frac{-2}{p}\right) (x - p/2) \sin \frac{n\pi}{p} x \, dx$$

$$\overset{IBP}{=} \frac{4}{p n\pi} \cos \frac{n\pi}{p} x \,(x - p/2) \Big|_0^p - \frac{4}{p n\pi} \int_0^p \cos \frac{n\pi}{p} x \, dx$$

$$= \frac{4}{pn\pi} \frac{p}{2}\left((-1)^n + 1\right) - \frac{4}{pn\pi} \frac{p}{n\pi} \sin \frac{n\pi x}{p} \Big|_{x=0}^{p}$$

$$= \frac{2}{n\pi}\left((-1)^n + 1\right) = \begin{cases} \frac{4}{n\pi} & \text{if } n \text{ is even} \\ 0 & \text{—— odd} \end{cases}$$

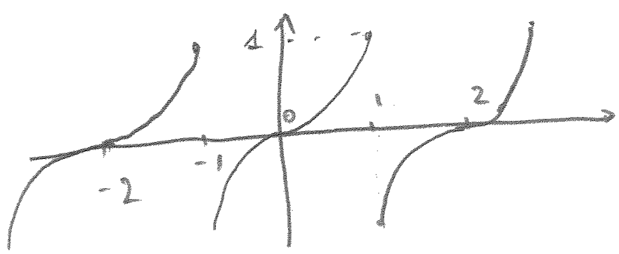
2.3.7 (cont'd)

Thus $\boxed{f(x) = \sum_{k=1}^{\infty} \frac{4}{2k\pi} \sin \frac{2k\pi}{p} x}$

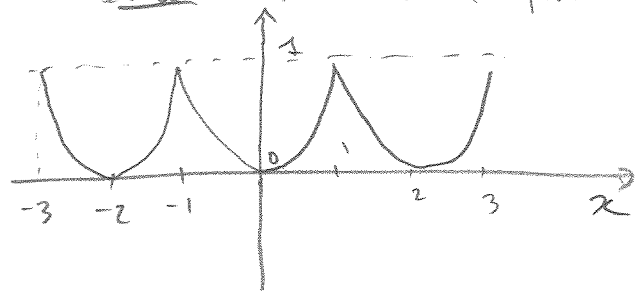
at the disc. pts the sine series converges to zero.

2.4.3 $f(x) = x^2 \quad 0 < x < 1$

(a) odd extension (2-per)



even extension (2-per)



Sine series

$b_n = 2\int_0^1 x^2 \sin n\pi x\, dx$

$\overset{IBP}{=} -2x^2 \frac{\cos n\pi x}{n\pi}\Big|_0^1 + \frac{4}{n\pi}\int_0^1 x \cos n\pi x\, dx$

$\overset{IBP}{=} -2\frac{(-1)^n}{n\pi} + \underbrace{\frac{4}{(n\pi)^2} x \sin n\pi x\Big|_0^1}_{=0} - \frac{4}{(n\pi)^2}\int_0^1 \sin n\pi x\, dx$

$= -2\frac{(-1)^n}{n\pi} + \frac{4}{(n\pi)^2}\frac{\cos n\pi x}{n\pi}\Big|_0^1$

$= -2\frac{(-1)^n}{n\pi} + \frac{4}{(n\pi)^3}\left((-1)^n - 1\right)$

$\Rightarrow \boxed{f(x) = \sum_{n=1}^{\infty}\left(-2\frac{(-1)^n}{n\pi} + \frac{4}{(n\pi)^3}\left((-1)^n - 1\right)\right)\sin n\pi x}$

See attached code & plot $x \in [0,1]$

Cosine series

$a_0 = \int_0^1 x^2\, dx = \frac{1}{3}$

$a_n = 2\int_0^1 x^2 \cos n\pi x\, dx$

$\overset{IBP}{=} 2x^2 \frac{\sin n\pi}{n\pi}\Big|_0^1 - \frac{4}{n\pi}\int_0^1 x \sin n\pi x\, dx$

$\overset{IBP}{=} \frac{4}{n\pi} x \frac{\cos n\pi x}{n\pi}\Big|_0^1 - \underbrace{\cancel{\frac{4}{(n\pi)^2}\int_0^1 \cos n\pi x\, dx}}_{=0}$

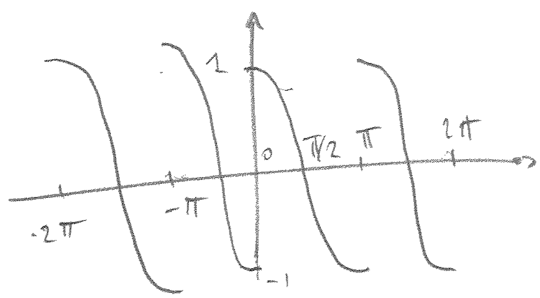
$= \frac{4}{(n\pi)^2}(-1)^n$

$\Rightarrow \boxed{f(x) = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{4}{(n\pi)^2}(-1)^n \cos n\pi x}$

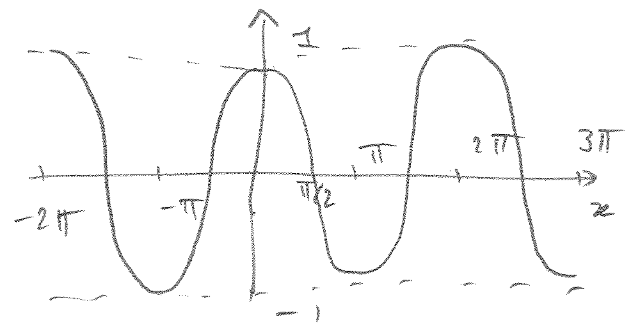
$x \in [0,1]$

$\boxed{2.4.6}$ $f(x) = \cos x \qquad 0 < x < \pi$

odd ext



even ext



$f(x) = \cos x$ (function itself.

Sine serie:

$$b_n = \frac{2}{\pi}\int_0^{\pi} \cos x \sin n\ x\, dx$$

$$= \frac{1}{\pi}\int_0^{\pi} \sin(n+1)x\, dx + \frac{1}{\pi}\int_0^{\pi} \sin(n-1)x\, dx$$

$$\overset{n\neq 1}{=} -\frac{1}{\pi(n+1)} \cos(n+1)x \Big|_0^{\pi}$$

$$- \frac{1}{\pi(n-1)} \cos(n-1)x \Big|_0^{\pi}$$

$$= \frac{-1}{\pi(n+1)}\left(\left((-1)^{n+1}-1\right) - \frac{1}{\pi(n-1)}\left((-1)^{n-1}-1\right)\right)$$

for $n \neq 1$

for $n=1$:

$$b_1 = \frac{2}{\pi}\int_0^{\pi} \cos x \sin x\, dx$$

$$= \frac{1}{\pi}\int_0^{\pi} \sin(2x)\, dx$$

$$= \frac{1}{2\pi} \cos 2x \Big|_0^{1}$$

$$= 0$$

$$\Rightarrow \boxed{b_n = \begin{cases} \frac{2}{\pi}\left(\frac{1}{n+1} + \frac{1}{n-1}\right) & n \text{ even} \\ 0 & n \text{ odd} \end{cases}}$$

```
% Math 3150
% Problem 2.4.3
thickLines(3); % remove if you don't have it in your system
figure(1); clf;
x = linspace(0,1,1000);


% loop over number of terms
Ns = [1,2,5,10,25];
cols={'g','m','c','r','k'};

% SINE SERIES
figure(1); clf;
% plot true function for refernce
hold on;
plot(x,x.^2);
for iN = 1:length(Ns),
 N=Ns(iN);

 % compute partial sine series
 s=zeros(size(x));
 for n=1:N,
  bn = -2*(-1)^n/(n*pi) + 4/(n*pi)^3 * ( (-1)^n-1);
  s=s+bn*sin(n*pi*x);
 end;

 % comparative plot
 plot(x,s,cols{iN});
end;%N
hold off;
xlabel('x');
title('problem 2.4.3, sine series');
filename='p2_4_3_sin.eps';
print('-depsc2',filename);
system(['epstopdf ' filename]);


% COSINE SERIES
figure(2); clf;
% plot true function for refernce
hold on;
plot(x,x.^2);
for iN = 1:length(Ns),
 N=Ns(iN);

 % compute partial cosine series
 a0 = 1/3;
 s=a0*ones(size(x));
 for n=1:N,
  an = 4/(n*pi)^2 * (-1)^n;
  s=s+an*cos(n*pi*x);
```

```
  end;

  % comparative plot
  plot(x,s,cols{iN});
end;%N
hold off;
xlabel('x');
axis([0,1,-0.1,1.2]);
title('problem 2.4.3, cosine series');
filename='p2_4_3_cos.eps';
print('-depsc2',filename);
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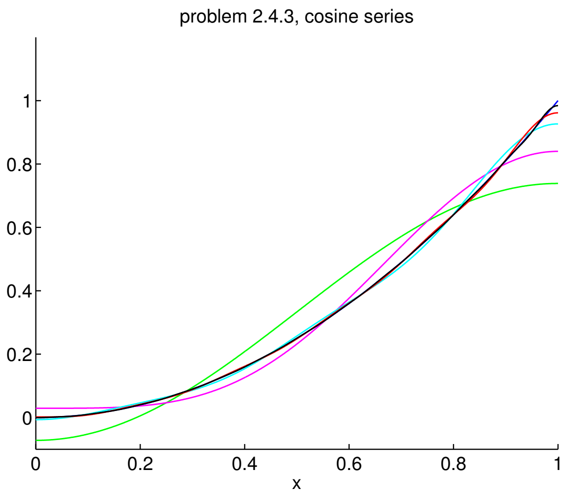

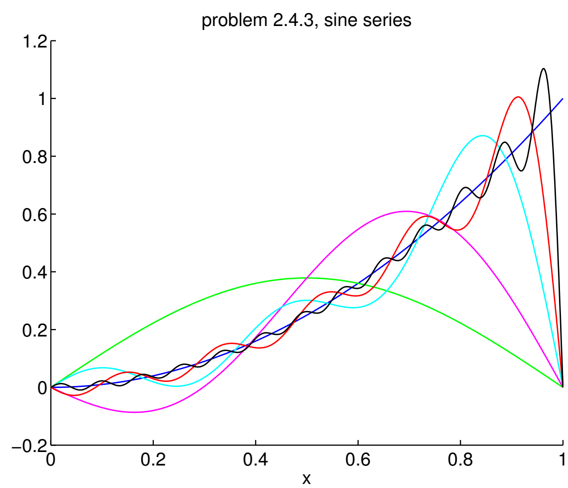

```
% MATH 3150 Fall 2008
% Problem 2.4.6

thickLines(3); % remove if you don't have it in your system
figure(1); clf;
x = linspace(0,pi,1000);

% plot true function for refernce
hold on;
plot(x,cos(x));
axis([0,pi,-1.2,1.2]);

% loop over number of terms
Ns = [1,2,5,10,25];
cols={'g','m','c','r','k'};
for iN = 1:length(Ns),
 N=Ns(iN);

 % compute partial Fourier series
 s=zeros(size(x));
 for k=1:N,
  bn = 4*(2*k)/pi/((2*k)^2-1);
  s=s+bn*sin(2*k*x);
 end;

 % comparative plot
 plot(x,s,cols{iN});
end;%N
hold off;
xlabel('x');
title('problem 2.4.6');
filename='p2_4_6.eps';
print('-depsc2',filename);
system(['epstopdf ' filename]);
```