MATH 3150 HW 2 Solutions

Problem 1: See code and plot. We should be able to see both right and left traveling solutions to WEQ.

Problem 2

$$\underline{x} = \begin{pmatrix} 4 \\ 2 \\ -2 \\ 1 \end{pmatrix}, \quad \underline{y} = \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$$

(a) $\|\underline{x}\| = \sqrt{16+4+4+1} = \sqrt{25} = 5$

$\|\underline{y}\| = \sqrt{1+1+1+1} = 2$

(b) $$\boxed{\cos\theta = \frac{(\underline{x},\underline{y})}{\|\underline{x}\|\,\|\underline{y}\|} = \frac{1}{10}\times(4-2-2-1) = -\frac{1}{10}}$$

(c) $$\boxed{\text{proj} = \frac{(\underline{x},\underline{y})}{(\underline{y},\underline{y})}\underline{y} = \frac{-1}{4}\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}}$$

Problem 3

$$\underline{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad \underline{v}_2 = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}, \quad \underline{v}_3 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

(a) This is an $\perp$ family of vectors since:

$(\underline{v}_1, \underline{v}_2) = -1 + 2 - 1 = 0$

$(\underline{v}_1, \underline{v}_3) = 1 + 0 - 1 = 0$

$(\underline{v}_2, \underline{v}_3) = -1 + 0 + 1 = 0$

(b)
$$\underline{x} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \sum_{j=1}^{3} \alpha_j \underline{v}_j$$

$$\alpha_1 = \frac{(\underline{x}, \underline{v}_1)}{(\underline{v}_1, \underline{v}_1)} = \frac{5}{6}$$

$$\alpha_2 = \frac{(\underline{x}, \underline{v}_2)}{(\underline{v}_2, \underline{v}_2)} = \frac{-2}{3}$$

$$\alpha_3 = \frac{(\underline{x}, \underline{v}_3)}{(\underline{v}_3, \underline{v}_3)} = \frac{1}{2}$$

(c)
$$\frac{5}{6}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} - \frac{2}{3}\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$= \begin{bmatrix} 5/6 + 4/6 + 3/6 \\ 10/6 - 4/6 + 0 \\ 5/6 + 4/6 - 3/6 \end{bmatrix} = \begin{bmatrix} 12/6 \\ 6/6 \\ 6/6 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \checkmark$$

Problem 4 (§ 2.1.6)

Consider the functions:

$$1 \quad \cos\frac{\pi}{p}x, \quad \cos\frac{2\pi}{p}x, \quad \ldots$$

$$\sin\frac{\pi}{p}x, \quad \sin\frac{2\pi}{p}x, \quad \ldots$$

(a) Common period is $2p$.

(b) We need to check this is an $\perp$ family.

Here are the different cases: $(m \neq n)$

$$\boxed{\left(\cos\frac{n\pi x}{p}, \cos\frac{m\pi x}{p}\right)} = \int_{-p}^{p} \cos\frac{n\pi x}{p}\cos\frac{m\pi x}{p}\,dx$$

$$= \frac{1}{2}\int_{-p}^{p}\left[\cos\left(\frac{(n+m)\pi x}{p}\right) + \cos\left(\frac{(n-m)\pi x}{p}\right)\right]dx$$

$$\overset{n\neq m}{=} \frac{1}{2}\left(\frac{1}{(n+m)}\frac{p}{\pi}\sin\left(\frac{(n+m)\pi x}{p}\right) + \frac{1}{(n-m)}\frac{p}{\pi}\sin\left(\frac{(n-m)\pi x}{p}\right)\right]_{-p}^{p}$$

$$= 0$$

$$\boxed{\left(\cos\frac{n\pi x}{p}, \cos\frac{n\pi x}{p}\right)} = \frac{1}{2}\int_{-p}^{p}\left(1 + \cos\frac{2n\pi x}{p}\right)dx = \frac{1}{2}\times 2p = p$$

$$\boxed{(1, 1)} = \int_{-p}^{p} 1\,dx = 2p$$

$$\boxed{(\sin mx, \sin nx)} = \frac{1}{2}\int_{-p}^{p}\left(\cos(m-n)x\frac{\pi}{p} - \cos(m+n)x\frac{\pi}{p}\right)dx$$

$$\overset{n\neq m}{=} \frac{1}{2}\left[\frac{p/\pi}{m-n}\sin(m-n)x\frac{\pi}{p} - \frac{p/\pi}{m+n}\sin(m+n)\frac{x\pi}{p}\right]_{-p}^{p}$$

$$= 0$$

$$\boxed{(\sin nx, \sin nx)} = \frac{1}{2}\int_{-p}^{p}\left(1 - \cos\frac{2n\,x\pi}{p}\right)dx = \frac{1}{2}2p = p$$

$$\boxed{(\sin mx, \cos nx)} = \frac{1}{2}\int_{-p}^{p}\left(\sin(m+n)\frac{x\pi}{p} + \sin(m-n)\frac{x\pi}{p}\right)dx$$

$$\overset{n\neq m}{=} \frac{1}{2}\left[-\frac{p/\pi}{m+n}\cos(m+n)\frac{x\pi}{p} - \frac{p/\pi}{m-n}\cos(m-n)\frac{x\pi}{p}\right]_{-p}^{p}$$

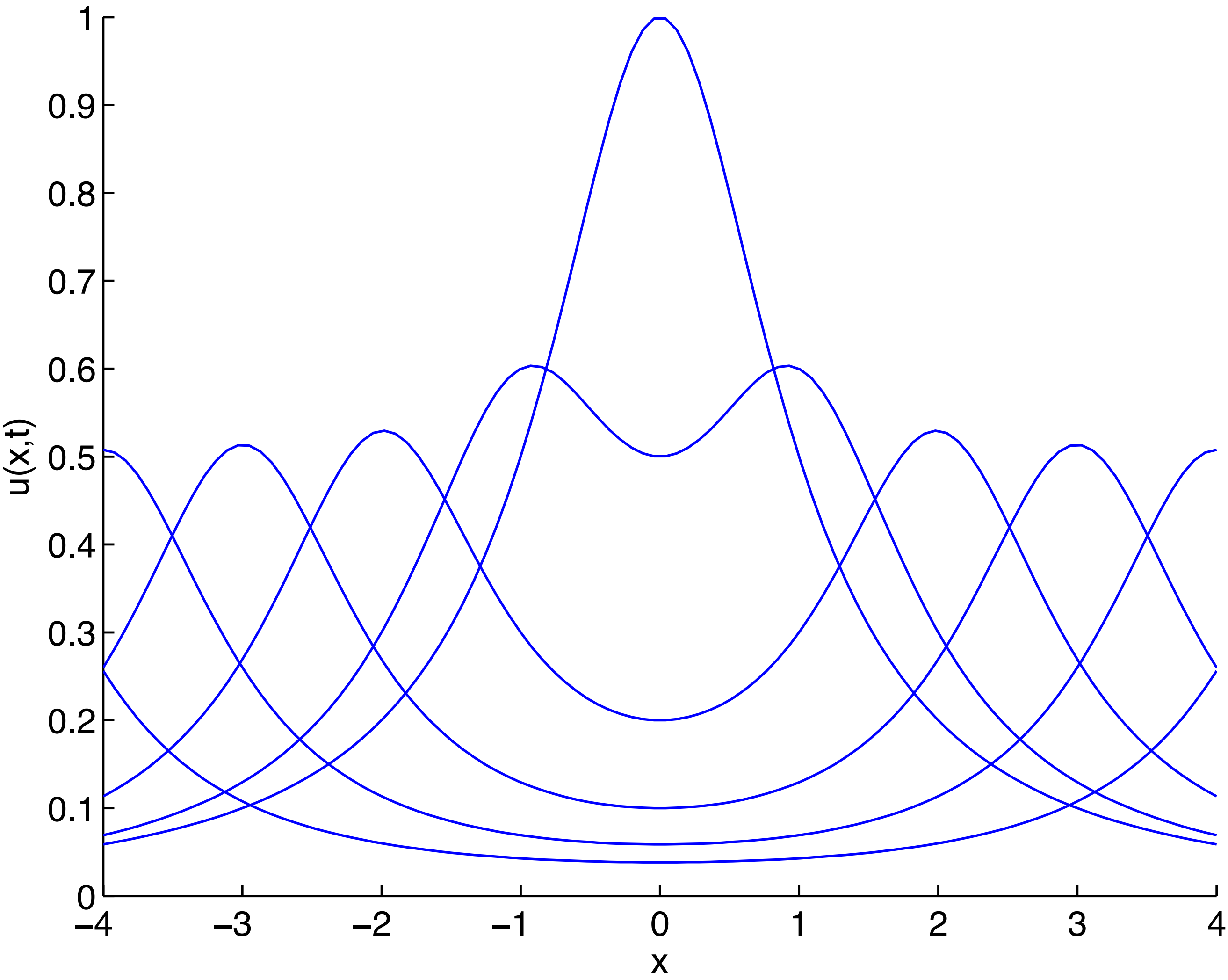
$$= 0$$

$$\boxed{(\sin nx, \cos nx)} = \frac{1}{2}\int_{-p}^{p}\sin\left(2n\frac{x\pi}{p}\right)dx = -\frac{p/\pi}{2n}\cos\left(\frac{2n x\pi}{p}\right)\Big|_{-p}^{p} = 0$$

```
% plots solution to wave equation using d'Alembert's method

x  = linspace(-4,4);

figure(1); clf;
hold on;
for t=0:5,
 plot( x, (1/2)* ( 1./(1+(x+t).^2) + 1./(1+(x-t).^2)) );
end;
hold off;
xlabel('x'); ylabel('u(x,t)');
```



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
u(x,t)
−4
−3
−2
−1
0
1
2
3
4
x