**Math 3150-4 – HW 2**
**due Wed Feb 1st in class**

**Problem 1.** A solution to the wave equation $u_{tt} = u_{xx}$ with initial conditions

$$u(x,0) = \frac{1}{1+x^2}, \ \frac{\partial u}{\partial t}(x,0) = 0, \ -\infty < x\infty,$$

is $u(x,t) = \frac{1}{2}(f(x+t)+f(x-t))$, with $f(x) = 1/(1+x^2)$. Plot snapshots of $u(x,t)$ at times $t = 0,1,2,3,4$ on the interval $[-5,5]$. **As in all problems involving plotting: include the code to generate the plot and a properly labeled plot**.

**Problem 2.** Consider the two vectors in $\mathbb{R}^4$,

$$\mathbf{x} = \begin{bmatrix} 4 \\ 2 \\ -2 \\ 1 \end{bmatrix} \text{ and } \mathbf{y} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}.$$

(a) Find $\|\mathbf{x}\|$ and $\|\mathbf{y}\|$ (i.e. their lengths)
(b) Find the cosine of the angle between the two vectors.
(c) Find the orthogonal projection of $\mathbf{x}$ onto $\mathbf{y}$.

**Problem 3.** Consider the following family of 3 vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}, \text{ and } \mathbf{v}_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}.$$

(a) Use inner products to verify that this is an orthogonal family of vectors.
(b) Use the orthogonal basis theorem (see class notes) to find the coefficients $\alpha_1, \alpha_2, \alpha_3$ in the expansion:

$$\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \sum_{j=1}^{3} \alpha_i \mathbf{v}_i.$$

sum index should be i

(c) Verify your answer.

**Problem 4.** Do problem 2.1.6 in your book. You need to compute the integrals by hand and show your work. (see discussion p21-22. We will see some tips on how to compute these integrals on Monday).