REVIEW SHEET FOR MIDTERM #2
MATH 2280-2

Chapter 4:

- Transform n-th order system to first order system
- Theorem 4.1.1: existence and uniqueness for linear systems
- Numerical methods for systems are basically vectorized versions of the numerical methods from 2.4-2.6.

Chapter 5:

- Know how to compute eigenvalues and eigenvectors of 2×2 and simple 3×3 matrices (e.g. triangular matrices).
- Know how to solve a 2×2 linear system or a simple 3×3 system.
- Computing general solutions to the homogeneous system $\mathbf{x}' = A\mathbf{x}$.
- Obtaining real solutions from conjugate pairs of eigenvalues and eigenvectors.
- Multiple eigenvalues. Algebraic (degree of eigenvalue in the characteristic polynomial) and geometric (number of linearly independent associated eigenvectors) multiplicity. How to find a chain of generalized eigenvectors and how to express the linearly independent solutions using such chains.
- Second order systems (spring mass systems)
- Notion of fundamental matrix solution and matrix exponential
- Three methods of computing matrix exponential: (1) fundamental matrix solution method, (2) power series, when matrix is nilpotent and (3) $A = aI + N$, where N is nilpotent.
- Finding a particular solution to non-homogeneous system with the method of undetermined coefficients.
- Derivation of the variation of parameters formulas 5.6 (22) and (29).

Chapter 6:

- Phase portraits. Being able to sketch solutions if only the tangent field is given.
- Notion of stability. Being able to decide on the stability and kind of critical point from the phase portrait.
- Stability of linear systems (Theorem 6.2.1) and nature of the point (saddle, spiral, node).
- Accurately sketch main features of saddle points (asymptotes of the hyperbolic trajectories, repulsive and attractive directions) and spirals (orientation).
- Finding the critical points (equillibria) of an autonomous system of DEs.
- Linearization of an autonomous system around a critical point systems. What can one say about the critical point from the linearization? (Theorem 6.2.2).