REVIEW SHEET FOR MIDTERM #1
MATH 2280-2

1. Theory

- Notion of DE, order of DE, interpretation as physical model
- Know how to sketch and interpret a slope field.
  - sketch solutions (Euler's method "by eye")
  - identify equilibriums and whether they are stable, unstable or neither.
- Existence and uniqueness theorem for first order DEs.
- Equilibrium solutions, critical points and phase diagram.
- Know how to interpret a bifurcation diagram.
- For $n-$th order **linear** DEs

$$L(y) = y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_1(x)y' + a_0(x)y = f(x) \tag{1}$$

  - Existence and uniqueness theorem
  - For the homogeneous DE $L(y) = 0$: Wronskian, linear independence of solutions and dimension of $\ker L$.
  - Any solution y to (1) can be written as $y = y_h + y_p$, where $y_h \in \ker L$ and $L(y_p) = f$ is a particular solution.
- For constant coefficient linear DE ($a_i(x) = a_i = \text{const}$):
  - relation between roots of characteristic polynomial and the solutions to $L(y) = 0$.
  - real roots, multiple roots and complex roots.
  - Euler's formula: $e^{i\theta} = \cos\theta + i\sin\theta$

2. Models

- Natural growth and decay: $y' = ky$. Applications to populations, interest rates, radioactive decay.
- Mixture problems $x' = r_i c_i - r_o c_o$, $c_o = x/V$.
- Population models: $P' = aP^2 + bP + c$. Logistic equation ($c = 0$), harvesting ($c \neq 0$). Identify limiting population, equilibriums, and doomsday/extinction scenarios.
- Acceleration/velocity models: linear resistance $mv' = -kv - mg$. Quadratic resistance <u>not included</u>.
- Pendulum $L\theta'' + g\theta = 0$
- Mechanical vibrations: $mx'' + cx' + kx = f(t)$
  - Identify regime :undamped, overdamped, underdamped, critically damped.
  - Identify properties of regime: natural frequency, pseudo-frequency, envelope, phase angle and amplitude.
  - Go from $A\cos\omega t + B\sin\omega t$ to $C\cos(\omega t - \alpha)$.

3. Methods

- **Integration** to find a {general,particular} solution to the simplest DE: $y'(x) = f(x)$.
- **Integrating factor method** for linear first order DEs: $y' + p(x)y = q(x)$.
- **Separation of variables** for separable DEs: $dy/dx = f(y)/g(x)$
- **Method of undetermined coefficients** for finding a particular solution to $L(y) = f$, where L is an $n-$th order linear differential operator with **constant coefficients**.
- **Method of variation of parameters:** <u>not included</u>.
- **Numerical methods**
  - Euler (1st order accurate) and Improved Euler (2nd order accurate). Runge Kutta <u>not included</u>.
  - Notion of order of accuracy