1. Theory

- Notion of DE, order of DE, interpretation as physical model
- Know how to sketch and interpret a slope field.
  - sketch solutions (Euler’s method “by eye”)
  - identify equilibriums and whether they are stable, unstable or neither.
- Existence and uniqueness theorem for first order DEs.
- Equilibrium solutions, critical points and phase diagram.
- Know how to interpret a bifurcation diagram.
- For \( n \)-th order linear DEs
  \[
  L(y) = y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_1(x)y' + a_0(x)y = f(x)
  \]
  - Existence and uniqueness theorem
  - For the homogeneous DE \( L(y) = 0 \): Wronskian, linear independence of solutions and dimension of ker \( L \).
  - Any solution \( y \) to (1) can be written as \( y = y_h + y_p \), where \( y_h \in \ker L \) and \( L(y_p) = f \) is a particular solution.
- For constant coefficient linear DE \( (a_i(x) = a_i = \text{const}) \):
  - relation between roots of characteristic polynomial and the solutions to \( L(y) = 0 \).
  - real roots, multiple roots and complex roots.
  - Euler’s formula: \( e^{i\theta} = \cos \theta + i \sin \theta \)

2. Models

- Natural growth and decay: \( y' = ky \). Applications to populations, interest rates, radioactive decay.
- Mixture problems \( x' = r_i c_i - r_o c_o, c_o = x/V \).
- Population models: \( P' = aP^2 + bP + c \). Logistic equation \( (c = 0) \), harvesting \( (c \neq 0) \). Identify limiting population, equilibriums, and doomsday/extinction scenarios.
- Acceleration/velocity models: linear resistance \( mv' = -kv - mg \). Quadratic resistance not included.
- Pendulum \( L\theta'' + g\theta = 0 \)
- Mechanical vibrations: \( mx'' + cx' + kx = f(t) \)
  - Identify regime: undamped, overdamped, underdamped, critically damped.
  - Identify properties of regime: natural frequency, pseudo-frequency, envelope, phase angle and amplitude.
  - Go from \( A\cos \omega t + B\sin \omega t \) to \( C\cos(\omega t - \alpha) \).

3. Methods

- Integration to find a \{general, particular\} solution to the simplest DE: \( y'(x) = f(x) \).
- Integrating factor method for linear first order DEs: \( y' + p(x)y = q(x) \).
- Separation of variables for separable DEs: \( dy/dx = f(y)/g(x) \)
- Method of undetermined coefficients for finding a particular solution to \( L(y) = f \), where \( L \) is an \( n\)-th order linear differential operator with constant coefficients.
- Method of variation of parameters: not included.
- Numerical methods
  - Euler (1st order accurate) and Improved Euler (2nd order accurate). Runge Kutta not included.
  - Notion of order of accuracy