Resonance (Example 9.4.2)

```
> unit:= (t,a,b) -> Heaviside(t-a) - Heaviside(t-b);
```

$$unit := (t, a, b) \rightarrow \mathrm{Heaviside}(t - a) - \mathrm{Heaviside}(t - b) \quad (1)$$

Square wave function

```
> F1 := t-> piecewise(0<t and t<=Pi, 10, Pi<t and t<=2*Pi,-10);
```

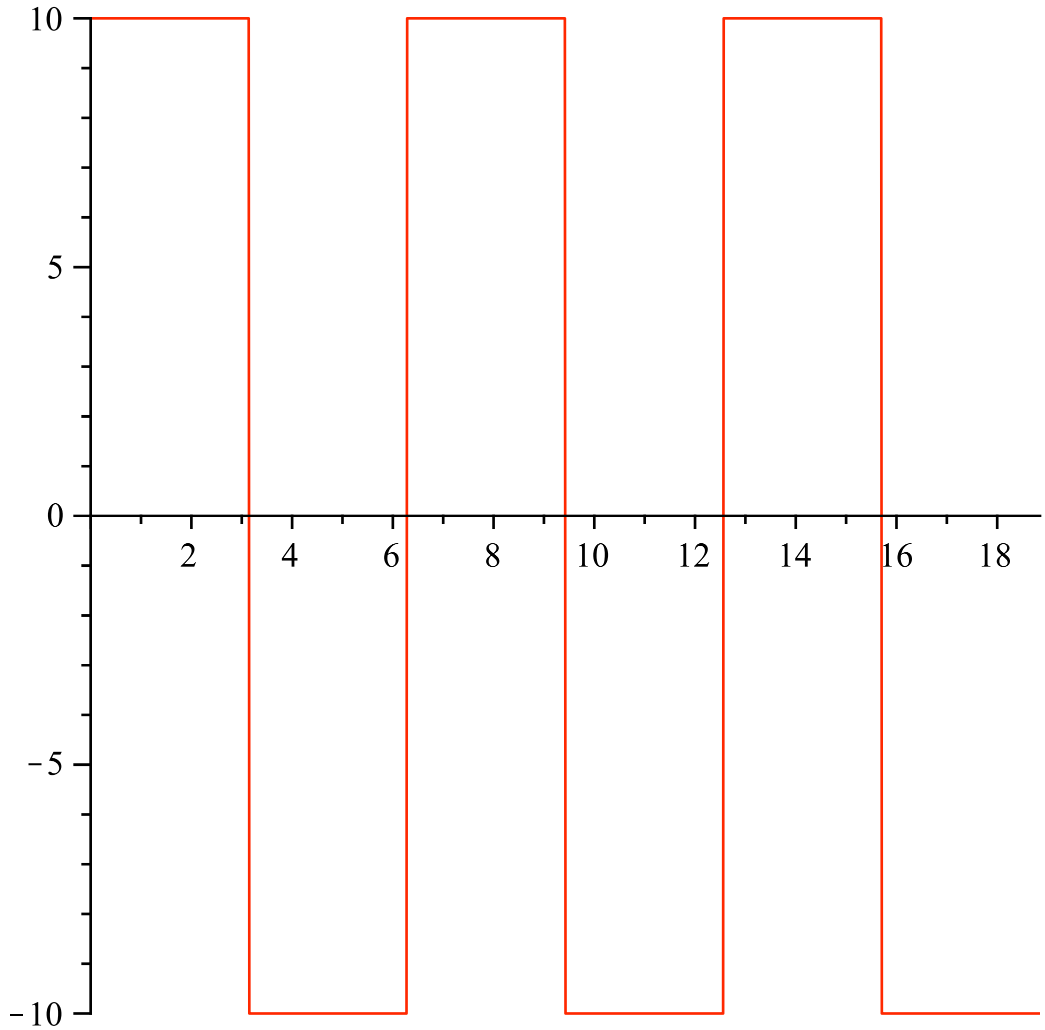
$$F1 := t \rightarrow piecewise(0 < t \text{ and } t \leq \pi, 10, \pi < t \text{ and } t \leq 2\pi, -10) \quad (2)$$

use Heaviside functions to get a few periods (at least for t>0, which is what we care for)

```
> F1per := t-> sum(Heaviside(t-2*Pi*n)*F1(t-2*Pi*n),n=0..5);
```

$$F1per := t \rightarrow \sum_{n=0}^{5} \mathrm{Heaviside}(t - 2\pi n)\, F1(t - 2\pi n) \quad (3)$$

```
> plot(F1per(t),t=0..6*Pi);
```




```
> b1 := n-> (1/Pi)*int(F1(t)*sin(n*t),t=0..2*Pi);
```

$$b1 := n \rightarrow \frac{\int_0^{2\pi} F1(t) \sin(n t)\, \mathrm{d}t}{\pi} \quad (4)$$

```
> b1(n) assuming integer;
```

$$-\frac{20\left(-1+(-1)^{n}\right)}{\pi n} \quad (5)$$

Sawtooth wave function

```
> F2 := t-> 10*t * unit(t,-Pi,Pi);
```

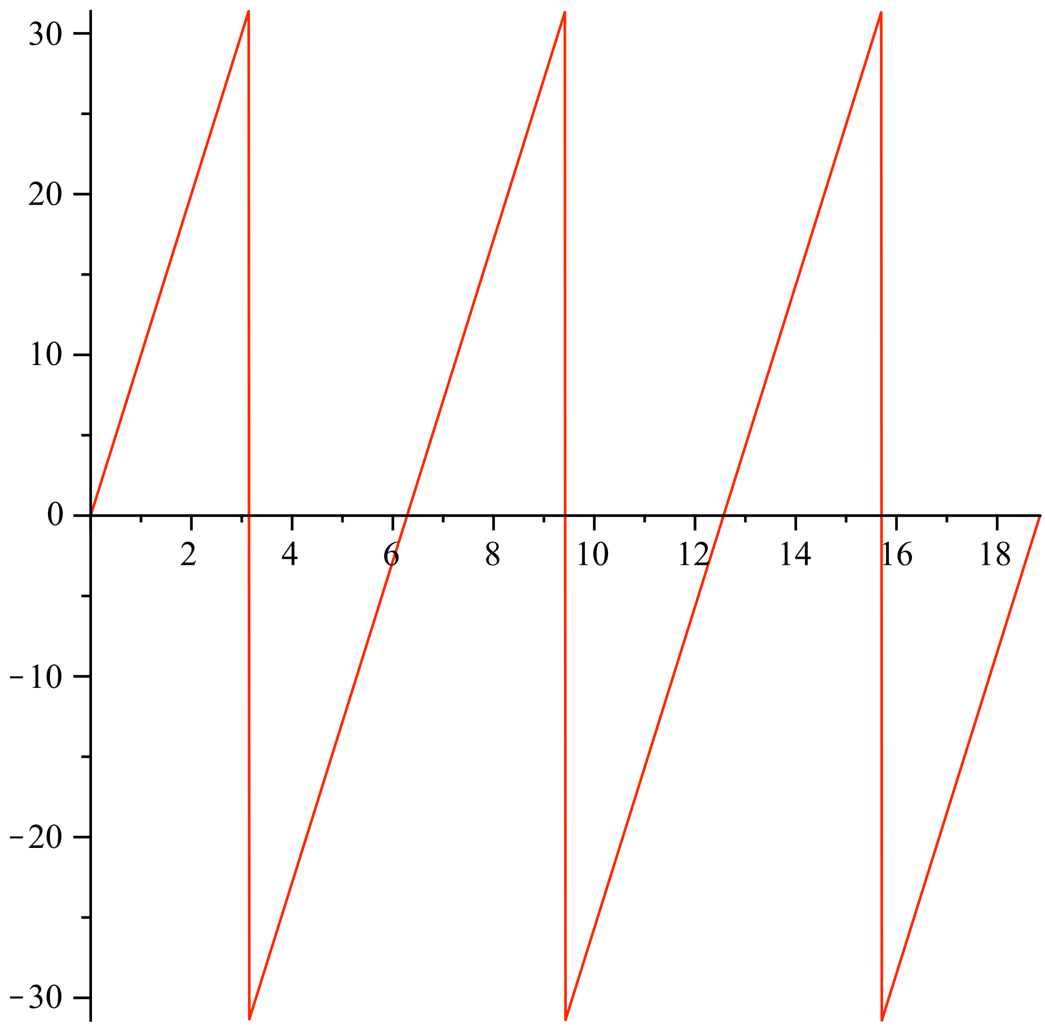
$$F2 := t \rightarrow 10\, t\, unit(t, -\pi, \pi) \quad (6)$$

Use Heaviside functions to get a few periods

```
> F2per := t-> sum(Heaviside(t-(2*n+1)*Pi)*F2(t-(2*n+2)*Pi),n=-1..5);
```

$$F2per := t \rightarrow \sum_{n=-1}^{5} \mathrm{Heaviside}(t-(2n+1)\pi)\, F2(t-(2n+2)\pi) \quad (7)$$

```
> plot(F2per(t),t=0..6*Pi);
```




```
> b2 := n-> (1/Pi)*int(F2(t)*sin(n*t),t=-Pi..Pi);
```

$$b2 := n \rightarrow \frac{\int_{-\pi}^{\pi} F2(t)\sin(n\,t)\, dt}{\pi} \quad (8)$$

```
> b2(n) assuming integer;
```

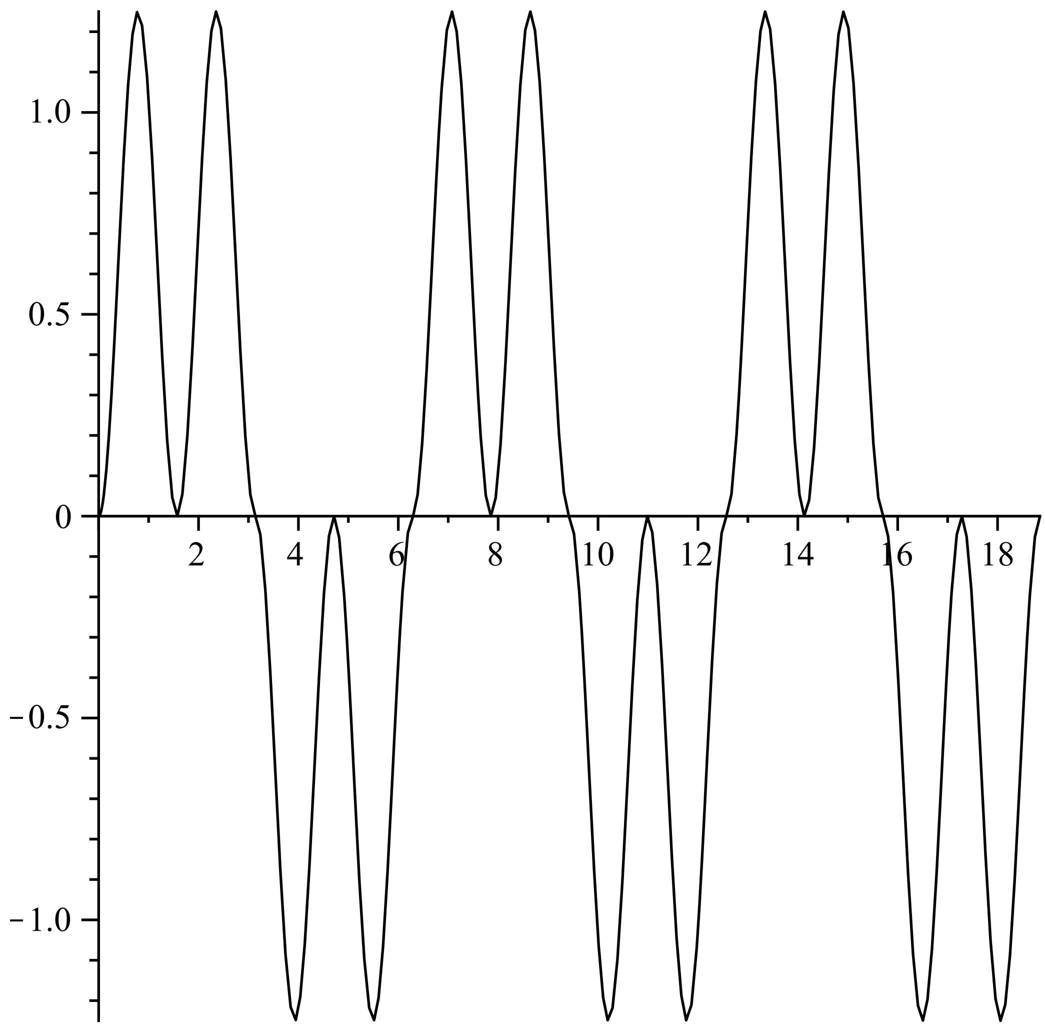
$$-\frac{20\,(-1)^n}{n} \tag{9}$$

Convolution solutions obtained using Laplace transform

```
> x1(t) := (1/8)*int(sin(4*(t-tau))*F1per(tau),tau=0..t):
> x2(t) := (1/8)*int(sin(4*(t-tau))*F2per(tau),tau=0..t):
```

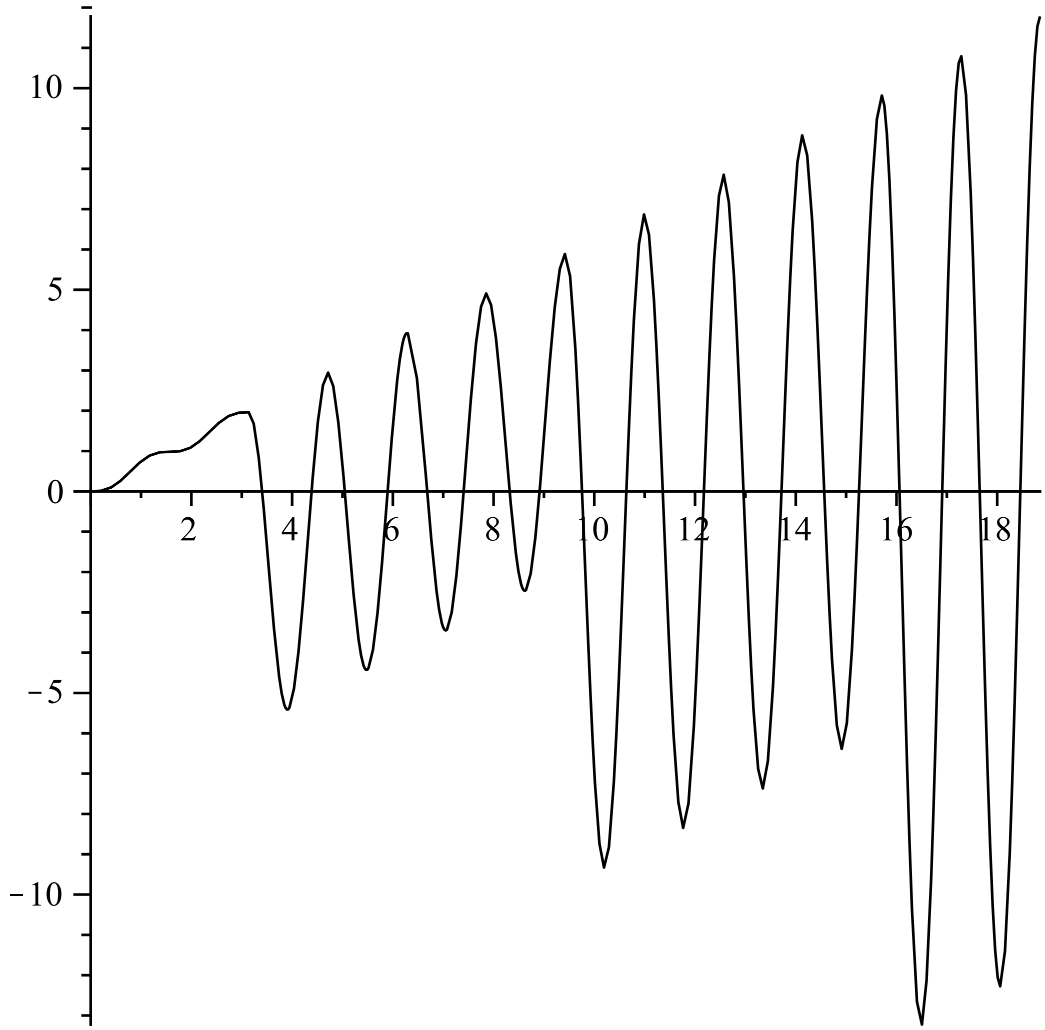
The displacement is periodic

```
> plot(x1(t),t=0..6*Pi,color=black);
```



The amplitude of the displacement increases with time (linearly) we have resonance.

```
> plot(x2(t),t=0..6*Pi,color=black);
```


10
5
0
-5
-10
2
4
6
8
10
12
14
16
18

>