

Math 2280-2 Computing Project #1

January 24 2008

This project is **due on Tuesday February 5 2008**. You may work in groups of 1–3 persons. Only one writeup per group should be submitted. Be sure to put your name(s) on the first page. Some problems may ask for algebraic manipulations to be carried out. You are free to either type them up in Maple or write them by hand and submit them with your solutions.

1. Complete investigation C on page 91 of your textbook. This involves finding the parameters in the logistic equation from the world population data given in Fig. 2.1.11 and using it to predict the world's population in 2025. You are encouraged to use and modify the Maple file we used in class, when we carried out a similar study on the US population. The maple worksheet can be found here:

http://www.math.utah.edu/~fguevara/math2280_s08/uspap.mws

For the following problems you are encouraged to use the Maple worksheets:

(W1) http://www.math.utah.edu/~fguevara/math2280_s08/denumsol.mw

(W2) http://www.math.utah.edu/~fguevara/math2280_s08/denumsol2.mw

(W3) http://www.math.utah.edu/~fguevara/math2280_s08/unstable.mw

Feel free to copy-paste and adapt the material in there.

2. We solve numerically on the interval $[0, 1]$, the same initial problem as in (W1), that is

$$\begin{aligned}\frac{dy}{dx} &= y \\ y(0) &= 1.\end{aligned}\tag{1}$$

- (a) Use the **improved Euler** algorithm, with $n = 100$ to approximate the solution on the x interval $[0, 1]$. Print your approximations at x_i values multiples of 0.1 along with the true values of the solution $y(x) = e^x$ evaluated at x_i and the error $|y(x_i) - y_i|$. The absolute value function in Maple is **abs(x)**.

- (b) Use the **Runge-Kutta** method with $n = 20$ for the same IVP (1). Print your approximations at x_i values multiples of 0.1 along with the true values of the solution $y(x) = e^x$ evaluated at x_i and the error $|y(x_i) - y_i|$.
 - (c) How many digits of precision for e you get in (a), (b), and with **unimproved Euler** with $n = 100$ (this is the example done in the worksheet).
3. Do problem 2.4.29 (page 122). This an example of a potential pitfall with numerical methods. If the solution has a singularity, it is possible to go past it if the step size is too large. It gets even worse since it is possible that the numerical method continues on a completely different solution!
4. We consider the Runge-Kutta second order method (RK2) given in pseudo-code:

RK2 method

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for  $i = 0 \dots n - 1$ 
     $k_1 \leftarrow f(x_i, y_i)$ 
     $k_2 \leftarrow f(x_{i+1/2}, y_i + hk_1/2)$ 
     $y_{i+1} \leftarrow y_i + hk_2$ 

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end for

The objective of this problem is to perform a comparative study of the accuracy of **RK2** and **improved Euler**. You may find (W2) helpful. Before answering the following questions make sure you have a working implementation of the RK2 method.

- (a) For both RK2 and improved Euler: Solve the same initial value problem $y' = 4/(1 + x^2)$, $y(0) = 0$ on the interval $[0, 1]$ for $n = 2, 4, 8, 16, 32, 64, 128$, displaying your approximation y_n , the exact solution $y(1) = 4 \arctan(1) = \pi$, and the error $|y_n - \pi|$ made in your approximation.
 - (b) For both RK2 and improved Euler: plot the data points $(\ln(n), \ln(|y_n - \pi|))$ and use linear regression to find the line that best fits these points. Produce a plot comparing the errors $(\ln(n), \ln(|y_n - \pi|))$ and their linear fits for both methods.
 - (c) What is the order of RK2 that you observe in this numerical experiment? Which method is more accurate for this example?
5. Do problem 2.5.30. This deals numerically with the crossbow example with quadratic air resistance—a nonlinear DE—with the same effort as with a linear DE.
6. Go to example 4 on pages 140–141. Work through and understand (for yourself) this example of a numerically unstable initial value problem. Recreate the data on the last two columns of Fig. 2.6.9: use Runge-Kutta to approximate the solution on $[0, 4]$ to the IVP (17) on p140, with $h = 0.05$, comparing with the values of the true solution at multiples of 0.4.

This is another potential pitfall of numerical methods: some problems are unstable in the sense that a small perturbation in the initial conditions can lead to very different results. Instability problems are basically why weather forecasts are not very accurate after a few days.