\[ p := 8 \]
\[ q := 2 \]

(1)

Diffusivity constant
\[ k := 1 + 0.1 \times q; \]

(2)

Length of the rod
\[ L := 100 + 10 \times p; \]

(3)

Initial temperature
\[ u_0 := 100; \]

(4)

Both ends of the rod are in an ice bath \((u(0,t) = u(L,t) = 0)\)

According to the analysis of Section 9.5 (separation of variables), the temperature at time \(t\) at the position \(x\) on the bar is given by the Fourier series (here we compute only the partial sum up to \(N\), \(N=50\) is good enough to get a good approximation)

\[
u := (x, t, N) \rightarrow 4 \times u_0 / \pi \times \sum_{j=1}^{N} \frac{1}{(2j-1)} \times \exp(-((2j-1)^2 \pi^2 k t / L^2)) \times \sin((2j-1) \pi x / L)
\]

(1.1)

The spatial distribution of the temperature on the rod after 30s is

\[ \text{plot} (u(x, 30, 50), x=0..L); \]
We can find the time at which the rod midpoint reaches 50 degrees:
\[M\text{plot}\{u(L/2, t, L), 50\}, \ t = 1. . 5000\]
We could estimate the time graphically or just use the built-in numerical methods in Maple

\[ T_{50} := \text{fsolve}(u(L/2, t, L) = 50, t = 0..5000); \]

\[ T_{50} = 2556.547907 \]  

The time in minutes and seconds is then:

\[ \text{printf}("%d: %.2f \n", \text{floor}(T_{50}/60), \text{frac}(T_{50}/60) * 60); \]

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End x=L is insulated while x=0 is ice bath

Here we use the analysis from problem 9.5.24 to find that the temperature on the rod is given by

\[ c := n-> \frac{4}{L} \int_{0}^{L} 100 \sin \left( \frac{1}{2} \frac{nq}{L} x \right) dx \]

\[ c := n-> \frac{2}{L} \left[ \int_{0}^{L} 100 \sin \left( \frac{1}{2} \frac{nq}{L} x \right) dx \right] \]  

\[ \text{(2.1)} \]

\[ \text{ubis} := (x, t, N) \rightarrow \sum c(2j + 1) \exp(-((2j + 1) Pi /2/L)^2k^2t) \sin((2j + 1) Pi /2/L), j = 0..N); \]

\[ \text{(2.2)} \]
Here is the typical temperature distribution after 3000s:

\[ u(x, 3000, 50) = u(x, L, t, L) = 50 \]

We can see that the maximum temperature of the rod is at the insulated end. We can find the time at which the rod midpoint reaches 50 degrees:

\[ \text{plot}(u(x, 3000, 50), x=0..L) \]

\[
ubis := (x, t, N) - \sum_{j=0}^{N} \frac{1}{4} (\frac{2\pi A}{2}) \frac{2q\sqrt{kt}}{L^2} \sin \left( \frac{1}{2} \left( \frac{2\pi j A}{L} \right) q x \right)
\]

(2.2)
We could estimate the time graphically or just use the built-in numerical methods in Maple

\[ T_{50} := \text{solve}(\text{ubis}(L, t, L) = 50, t=0..30000) ; \]

\[ T_{50} = 10226.19162 \]

The time in minutes and seconds is then:

\[ \text{printf}("%d: %.2f
", \text{floor}(T_{50}/60), \text{frac}(T_{50}/60)*60) ; \]

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