```
> p:=8;  q:=2;
```

$$p := 8$$
$$q := 2 \quad (1)$$

Diffusivity constant

```
> k:=1+0.1*q;
```

$$k := 1.2 \quad (2)$$

Length of the rod

```
> L := 100+10*p;
```

$$L := 180 \quad (3)$$

Initial temperature

```
> u0 := 100;
```

$$u0 := 100 \quad (4)$$

Both ends of the rod are in an ice bath (u(0,t)=u(L,t)=0)

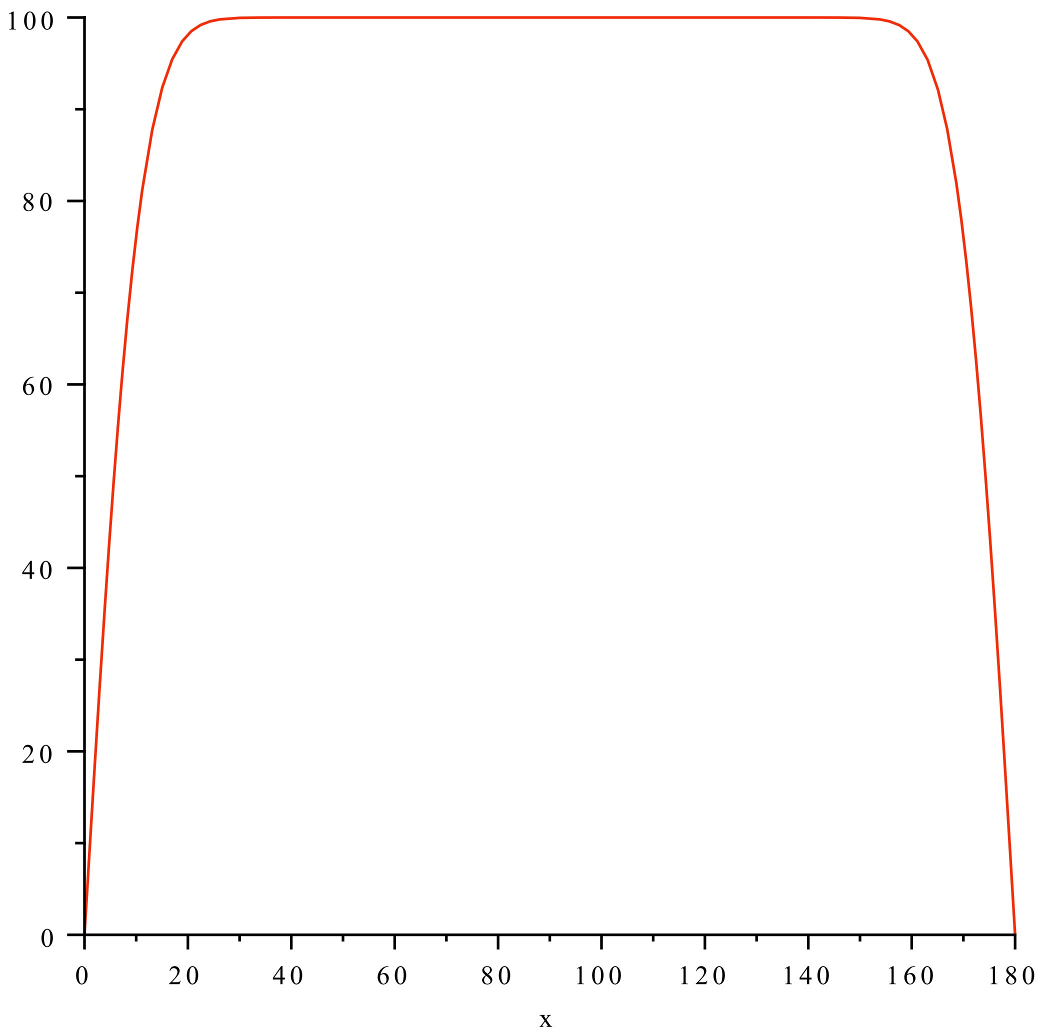
According to the analysis of Section 9.5 (separation of variables), the temperature at time t at the position x on the bar is given by the Fourier series (here we compute only the partial sum up to N, N=50 is good enough to get a good approximation)

```
> u := (x,t,N) -> 4*u0/Pi * sum(1/(2*j-1) * exp(-(2*j-1)^2*
    Pi^2*k*t/L^2) * sin((2*j-1)*Pi*x/L) , j=1..N);
```

$$u := (x, t, N) \to \frac{4\,u0 \left(\sum_{j=1}^{N} \frac{e^{-\frac{(2j-1)^2 \pi^2 k t}{L^2}} \sin\left(\frac{(2j-1)\pi x}{L}\right)}{2j-1} \right)}{\pi} \quad (1.1)$$

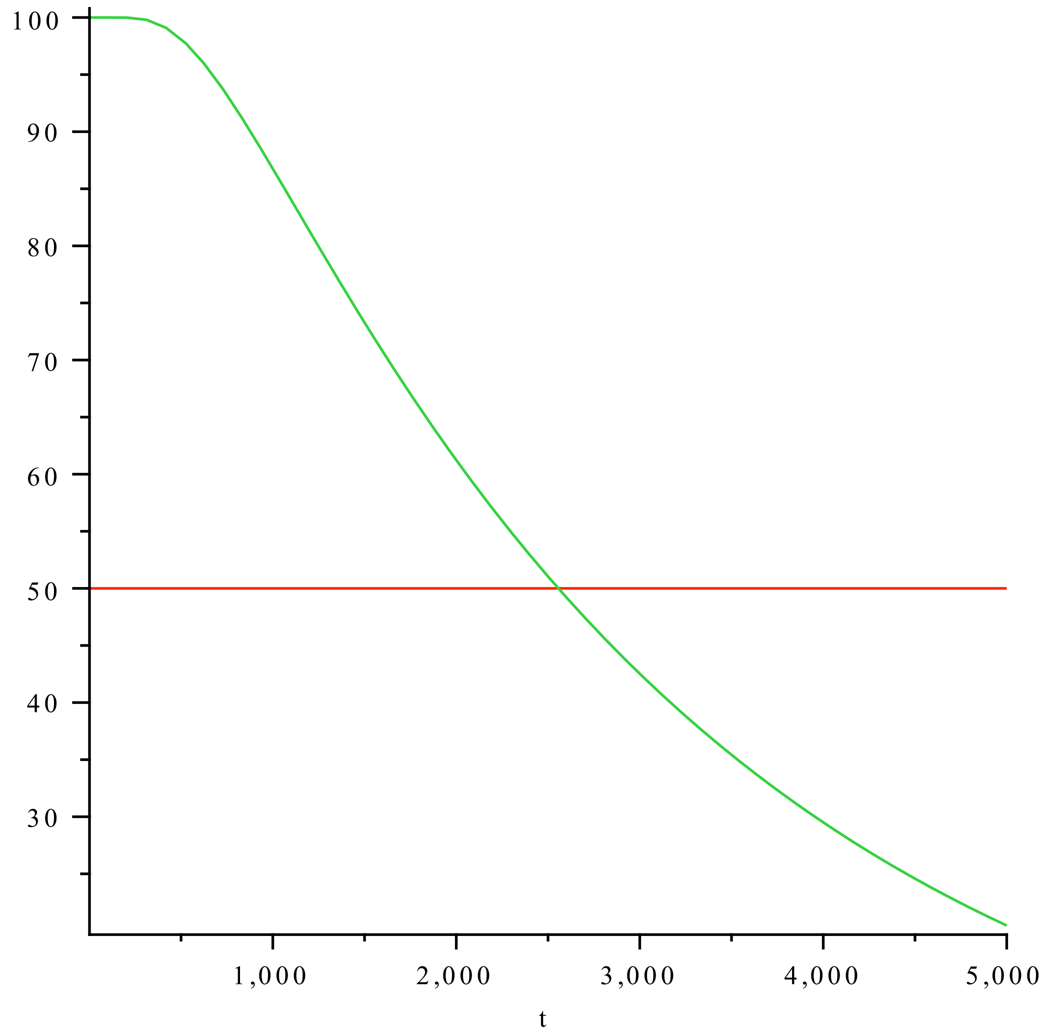
The spatial distribution of the temperature on the rod after 30s is

```
> plot(u(x,30,50), x=0..L);
>
```



We can find the time at which the rod midpoint reaches 50 degrees:

```
plot({u(L/2,t,L),50}, t=1..5000);
```



We could estimate the time graphically or just use the buit-in numerical methods in Maple

```
T50 := fsolve(u(L/2,t,L) = 50 , t=0..5000);
```

$$T50:=2556.547907 \quad (1.2)$$

The time in minutes and seconds is then:

```
printf("%d:%.2f\n",floor(T50/60), frac(T50/60)*60);
42:36.55
```

End x=L is insulated while x=0 is in ice bath

Here we use the anaylsis from problem 9.5.24 to find that the temperature on the rod is given by

```
c:= n-> (2/L)*int(100*sin(n*Pi*x/2/L),x=0..L);
```

$$c := n \to \frac{2\left(\int_0^L 100 \sin\left(\frac{1}{2}\,\frac{n q x}{L}\right) dx\right)}{L} \quad (2.1)$$

```
ubis := (x,t,N) -> sum(c(2*j+1)*exp(-((2*j+1)*Pi/2/L)^2*k*t)*
    sin((2*j+1)*Pi*x/2/L),j=0..N);
```

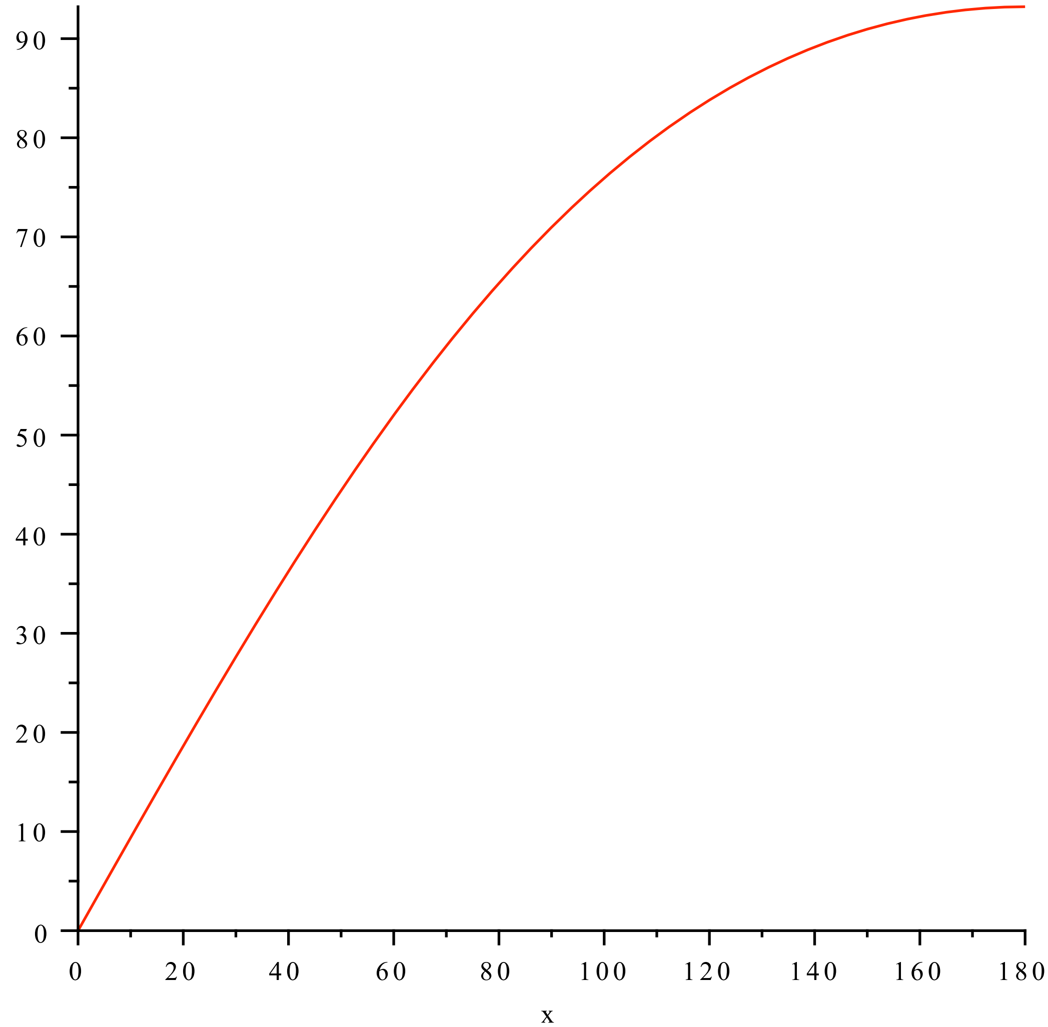
(2.2)

$$ubis := (x, t, N) \rightarrow \sum_{j=0}^{N} c(2\,j+1)\, e^{-\frac{1}{4}\frac{(2j+1)^2 \pi^2 k t}{L^2}} \sin\left(\frac{1}{2}\,\frac{(2\,j+1)\,\pi\,x}{L}\right) \qquad (2.2)$$

>

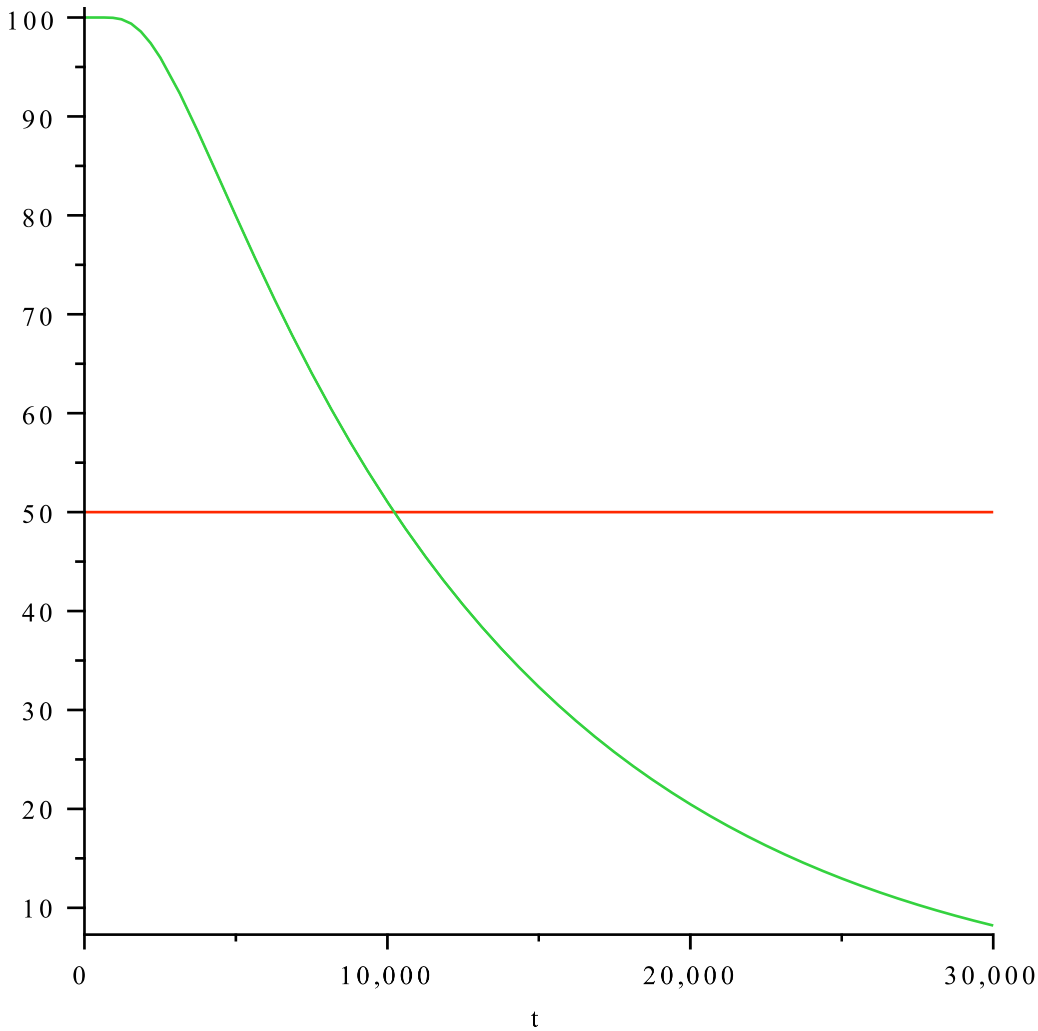
Here is the typical temperature distribution after 3000s

```
> plot(ubis(x, 3000, 50), x=0..L);
```



We can see that the maximum temperature of the rod is at the insulated end. We can find the time at which the rod midpoint reaches 50 degrees:

```
> plot({ubis(L, t, L), 50}, t=1..30000);
```



We could estimate the time graphically or just use the buit-in numerical methods in Maple

```
> T50 := fsolve(ubis(L,t,L) = 50 , t=0..30000);
```

$$T50 := 10226.19162 \quad (2.3)$$

The time in minutes and seconds is then:

```
> printf("%d:%.2f\n",floor(T50/60), frac(T50/60)*60);
170:26.19
>
```