Project 2, Application 9.3

```
> unit := (t,a,b) -> Heaviside(t-a) - Heaviside(t-b);
```

unit := (t, a, b) → Heaviside(t I a) I Heaviside(t I b) (1)

Define the Fourier coefficients for even (a(n)) or odd (b(n)) extensions and the corresponding Fourier series

```
> a := (f,n) -> (2/Pi)*int(f(t)*cos(n*t),t=0..Pi);
  b := (f,n) -> (2/Pi)*int(f(t)*sin(n*t),t=0..Pi);
  cossum := (f,N,t) -> a(f,0)/2 + sum(a(f,n)*cos(n*t),n=1..N);
  sinsum := (f,N,t) -> sum(b(f,n)*sin(n*t),n=1..N);
```

$$a := (f, n) \rightarrow \frac{2\left(\int_0^q f(t)\cos(n\,t)\,dt\right)}{q}$$

$$b := (f, n) \rightarrow \frac{2\left(\int_0^q f(t)\sin(n\,t)\,dt\right)}{q}$$

$$cossum := (f, N, t) \rightarrow \frac{1}{2}\,a(f, 0)\ A \underset{n=1}{\overset{N}{<}} a(f, n)\cos(n\,t)$$

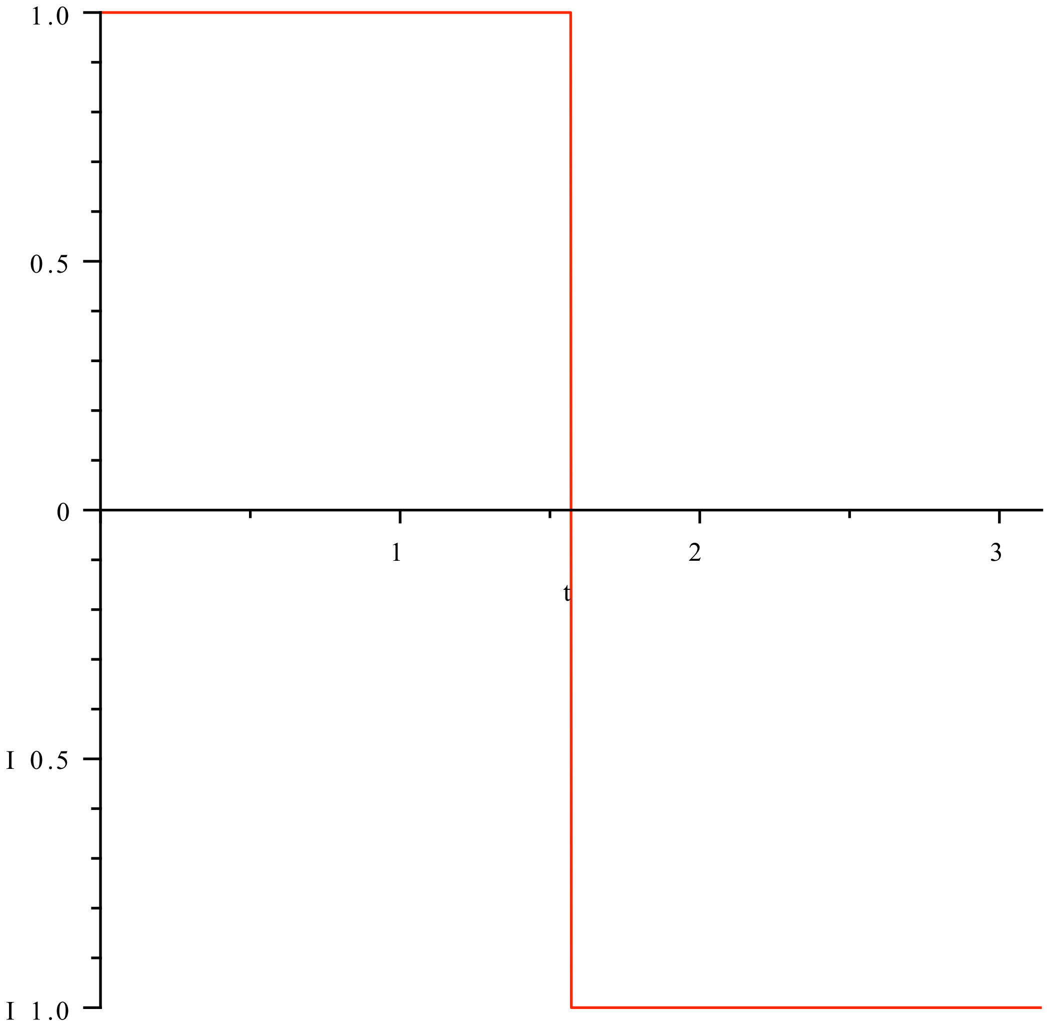
$$sinsum := (f, N, t) \rightarrow \underset{n=1}{\overset{N}{<}} b(f, n)\sin(n\,t)$$ (2)

Function from fig 9.3.9

```
> f1 := t -> unit(t,0,Pi/2) - unit(t,Pi/2,Pi);
```

$$f1 := t \rightarrow unit\left(t, 0, \frac{1}{2}\,q\right) I\ unit\left(t, \frac{1}{2}\,q, q\right)$$ (1.1)

```
> plot(f1(t),t=0..Pi);
```




```
M cossum(f1, 50, t);
```

$$\frac{4}{29}\,\frac{\cos(29\,t)}{q}\;I\;\frac{4}{31}\,\frac{\cos(31\,t)}{q}\;A\;\frac{4}{5}\,\frac{\cos(5\,t)}{q}\;A\;\frac{4}{25}\,\frac{\cos(25\,t)}{q}\;I\;\frac{4}{11}\,\frac{\cos(11\,t)}{q} \quad (1.2)$$
$$A\;\frac{4}{13}\,\frac{\cos(13\,t)}{q}\;A\;\frac{4}{9}\,\frac{\cos(9\,t)}{q}\;A\;\frac{4}{45}\,\frac{\cos(45\,t)}{q}\;I\;\frac{4}{7}\,\frac{\cos(7\,t)}{q}$$
$$I\;\frac{4}{39}\,\frac{\cos(39\,t)}{q}\;A\;\frac{4}{41}\,\frac{\cos(41\,t)}{q}\;I\;\frac{4}{43}\,\frac{\cos(43\,t)}{q}\;I\;\frac{4}{35}\,\frac{\cos(35\,t)}{q}$$
$$A\;\frac{4\cos(t)}{q}\;I\;\frac{4}{3}\,\frac{\cos(3\,t)}{q}\;I\;\frac{4}{15}\,\frac{\cos(15\,t)}{q}\;A\;\frac{4}{17}\,\frac{\cos(17\,t)}{q}\;I\;\frac{4}{19}\,\frac{\cos(19\,t)}{q}$$
$$A\;\frac{4}{21}\,\frac{\cos(21\,t)}{q}\;I\;\frac{4}{23}\,\frac{\cos(23\,t)}{q}\;I\;\frac{4}{27}\,\frac{\cos(27\,t)}{q}\;A\;\frac{4}{33}\,\frac{\cos(33\,t)}{q}$$
$$A\;\frac{4}{37}\,\frac{\cos(37\,t)}{q}\;I\;\frac{4}{47}\,\frac{\cos(47\,t)}{q}\;A\;\frac{4}{49}\,\frac{\cos(49\,t)}{q}$$

The pattern here is:

```
M f1bis := (t, N) -> (4/Pi)*sum((-1)^k * cos((2*k+1)*t)/(2*k+1),
  k=0..N);
```

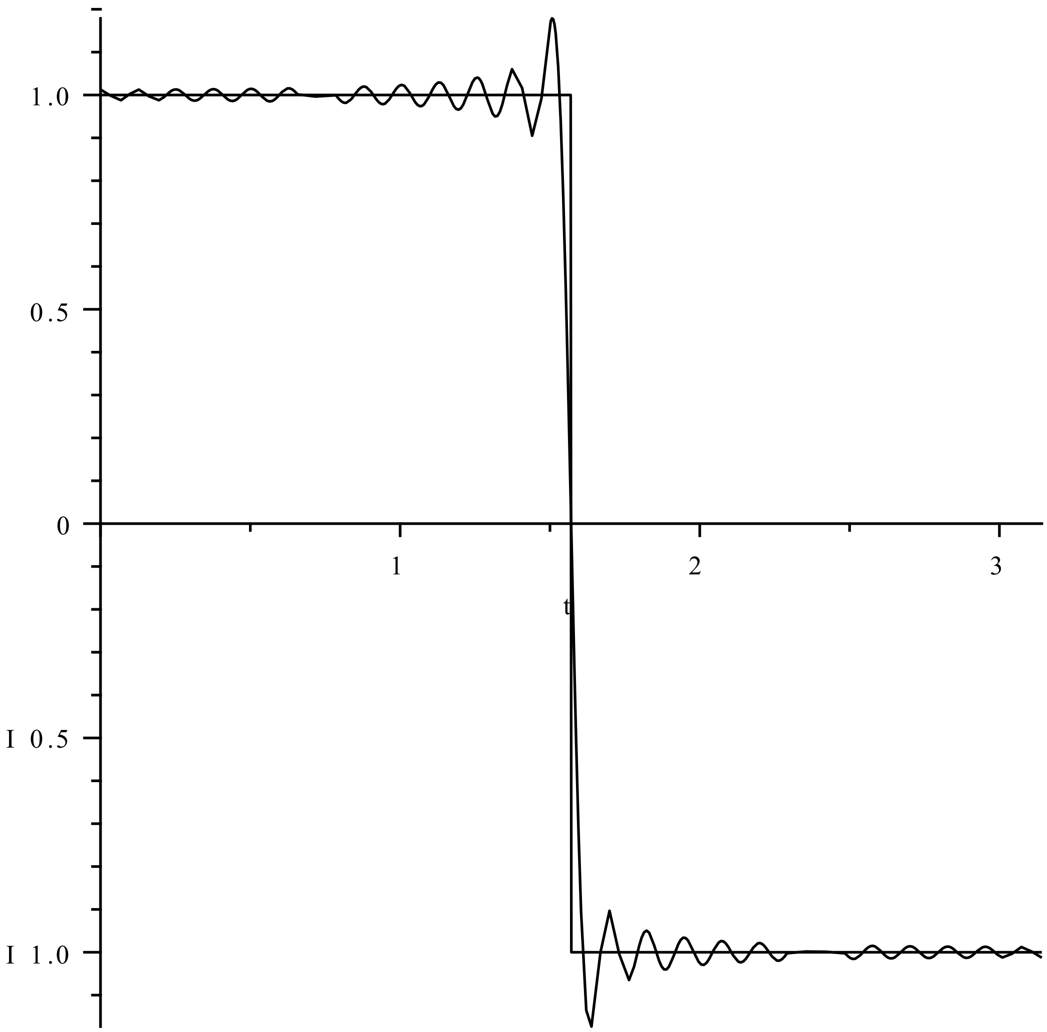
$$f1bis := (t, N) \to \frac{4 \left(\sum_{k=0}^{N} \frac{(-1)^k \cos((2k+1)\, t)}{2k+1} \right)}{\pi} \tag{1.3}$$

Just checking:

```
> simplify(f1bis(t,24)-cossum(f1,50,t));
```

$$0 \tag{1.4}$$

```
> plot({cossum(f1,50,t),f1(t)},t=0..Pi,color=black);
```

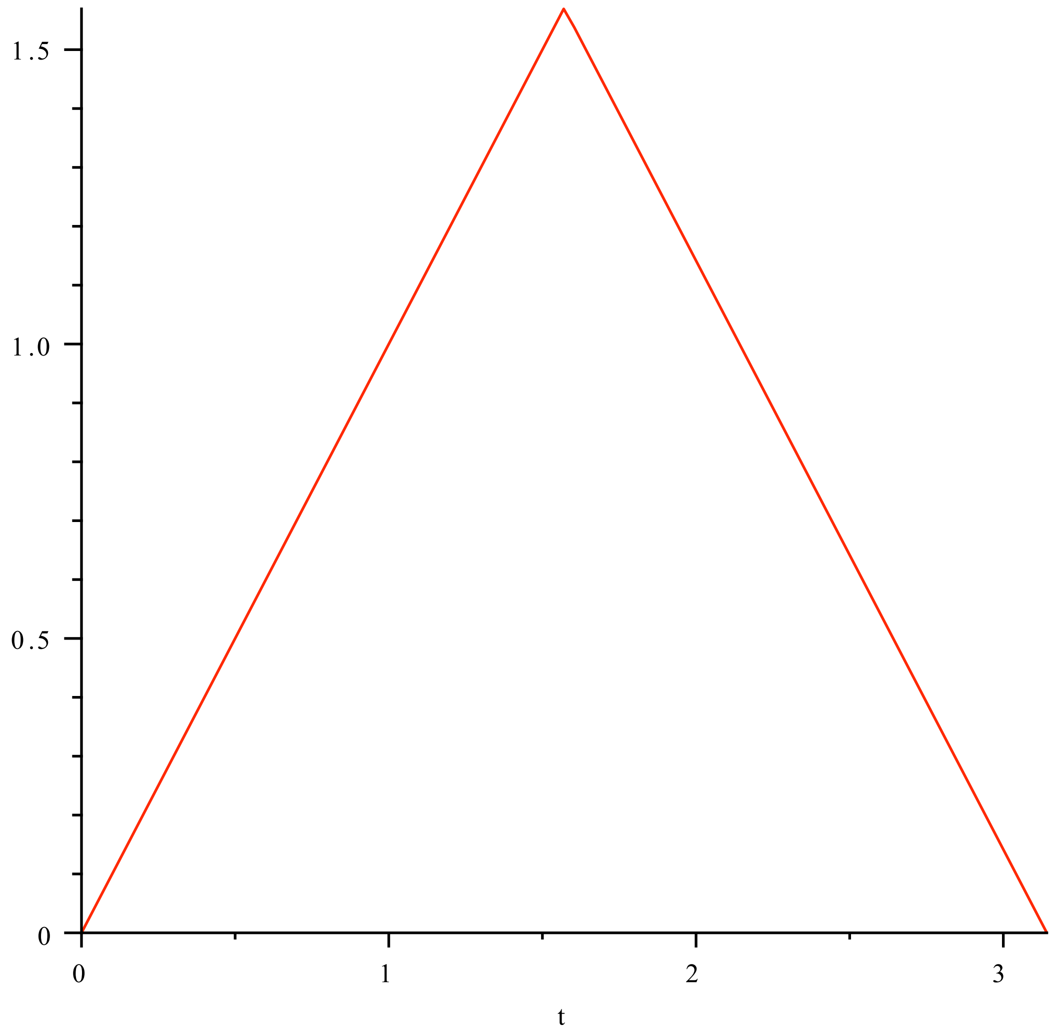


Functions from fig 9.2.4 and 9.3.10

```
> f2 := t -> t*unit(t,0,Pi/2) + (Pi-t)*unit(t,Pi/2,Pi);
```

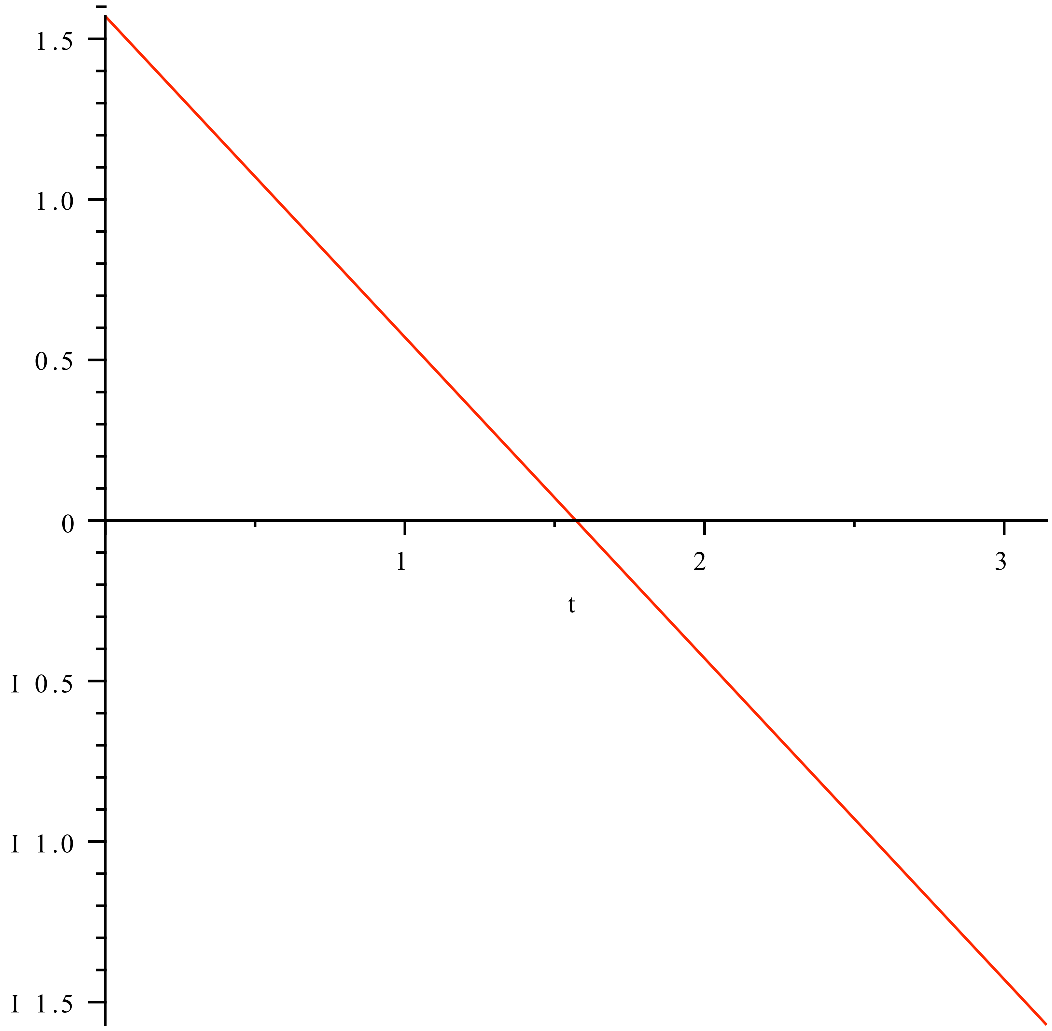
$$f2 := t \to t\, \mathrm{unit}\left(t, 0, \frac{1}{2}\pi\right) + (\pi - t)\, \mathrm{unit}\left(t, \frac{1}{2}\pi, \pi\right) \tag{2.1}$$

```
> plot(f2(t),t=0..Pi);
```



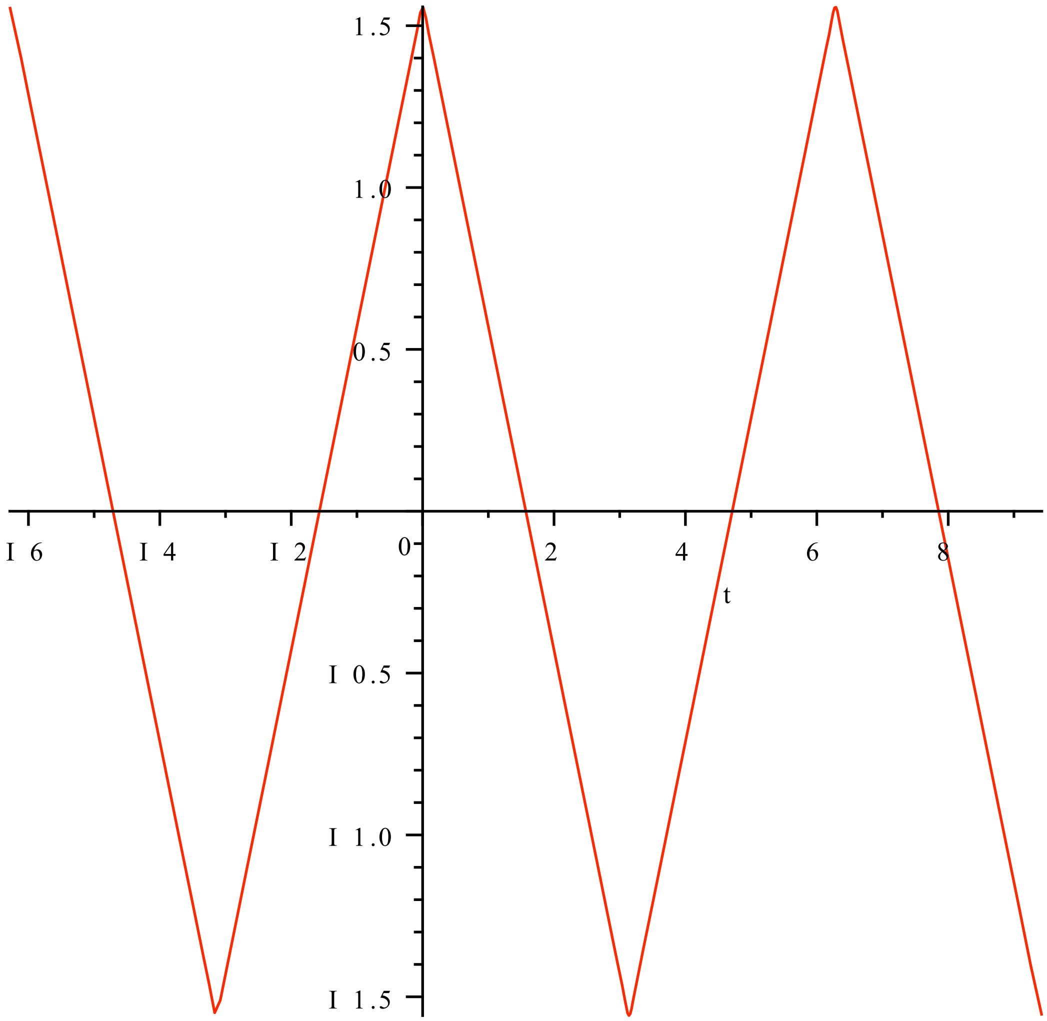

```
M f3 := t -> (Pi/2-t); plot(f3(t),t=0..Pi);
```

$$f3 := t\text{-}\ \frac{1}{2}\,\text{qI}\ \ t$$



The EVEN extension (fig 9.3.10)

```
> plot(cossum(f3,50,t),t=-2*Pi..3*Pi); cossum(f3,50,t);
```



$$\frac{4}{841}\,\frac{\cos(29\,t)}{q} \;A\; \frac{4}{961}\,\frac{\cos(31\,t)}{q} \;A\; \frac{4}{25}\,\frac{\cos(5\,t)}{q} \;A\; \frac{4}{625}\,\frac{\cos(25\,t)}{q}$$
$$A\; \frac{4}{121}\,\frac{\cos(11\,t)}{q} \;A\; \frac{4}{169}\,\frac{\cos(13\,t)}{q} \;A\; \frac{4}{81}\,\frac{\cos(9\,t)}{q} \;A\; \frac{4}{2025}\,\frac{\cos(45\,t)}{q}$$
$$A\; \frac{4}{49}\,\frac{\cos(7\,t)}{q} \;A\; \frac{4}{1521}\,\frac{\cos(39\,t)}{q} \;A\; \frac{4}{1681}\,\frac{\cos(41\,t)}{q} \;A\; \frac{4}{1849}\,\frac{\cos(43\,t)}{q}$$
$$A\; \frac{4}{1225}\,\frac{\cos(35\,t)}{q} \;A\; \frac{4\cos(t)}{q} \;A\; \frac{4}{9}\,\frac{\cos(3\,t)}{q} \;A\; \frac{4}{225}\,\frac{\cos(15\,t)}{q}$$
$$A\; \frac{4}{289}\,\frac{\cos(17\,t)}{q} \;A\; \frac{4}{361}\,\frac{\cos(19\,t)}{q} \;A\; \frac{4}{441}\,\frac{\cos(21\,t)}{q} \;A\; \frac{4}{529}\,\frac{\cos(23\,t)}{q}$$
$$A\; \frac{4}{729}\,\frac{\cos(27\,t)}{q} \;A\; \frac{4}{1089}\,\frac{\cos(33\,t)}{q} \;A\; \frac{4}{1369}\,\frac{\cos(37\,t)}{q} \;A\; \frac{4}{2209}\,\frac{\cos(47\,t)}{q}$$
$$A\; \frac{4}{2401}\,\frac{\cos(49\,t)}{q} \qquad (2.2)$$

Here the pattern repeats skips every other even number in the cos summation (it repeats every 4)

```
M f3e := (t,N) -> (4/Pi)*sum(cos((2*k+1)*t)/((2*k+1)^2), k=0..N)
```

;

$$f3e := (t, N) \to \frac{4 \left(\sum_{k=0}^{N} \frac{\cos((2k+1)\,t)}{(2k+1)^2} \right)}{\pi} \qquad (2.3)$$

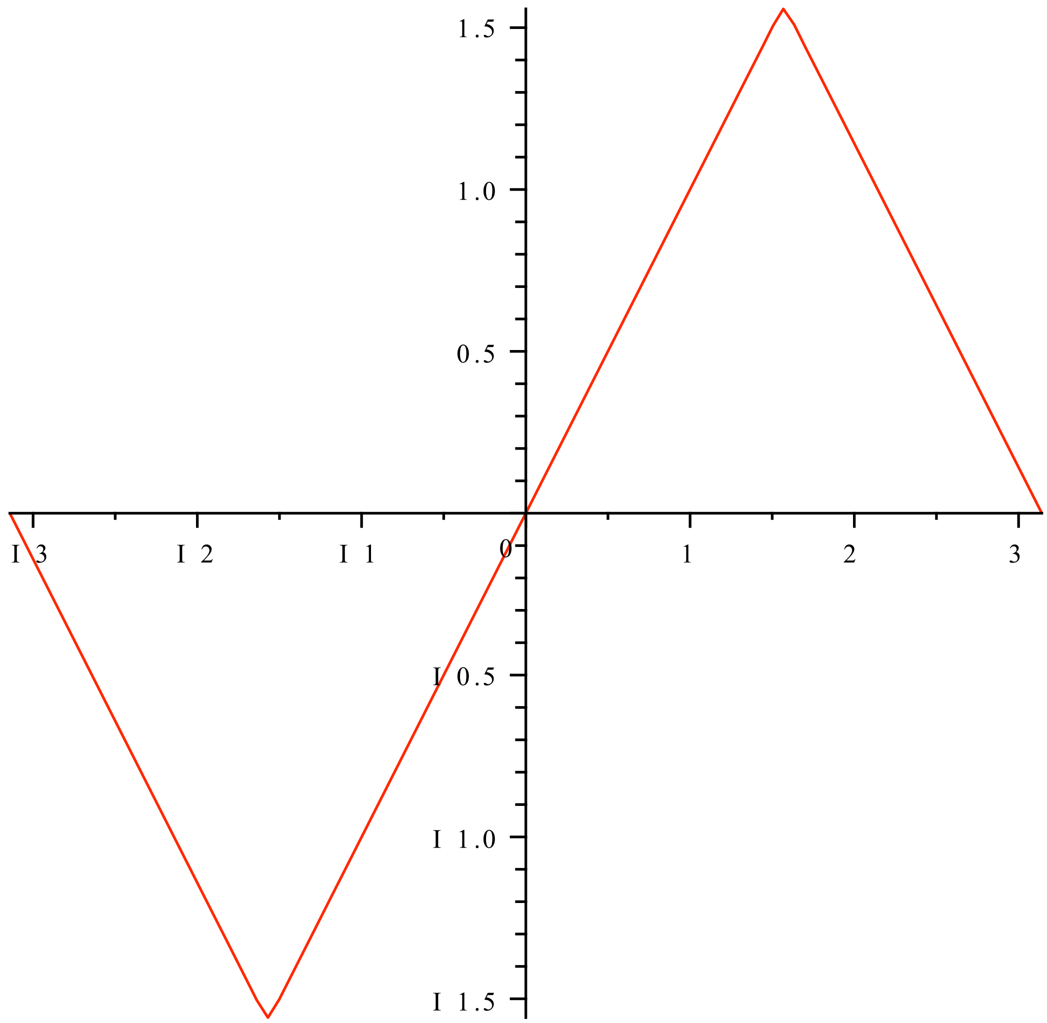
Checking

```
> simplify(f3e(t, 24)-cossum(f3, 50, t));
```

$$0 \qquad (2.4)$$

The ODD extension (fig 9.2.4)

```
> plot(sinsum(f2, 50, t), t=-Pi..Pi);  sinsum(f2, 50, t);
```



$$-\frac{4}{225}\,\frac{\sin(15\,t)}{\pi} - \frac{4}{2209}\,\frac{\sin(47\,t)}{\pi} + \frac{4}{2025}\,\frac{\sin(45\,t)}{\pi} - \frac{4}{1521}\,\frac{\sin(39\,t)}{\pi} + \frac{4}{1681}\,\frac{\sin(41\,t)}{\pi} - \frac{4}{1849}\,\frac{\sin(43\,t)}{\pi} + \frac{4}{81}\,\frac{\sin(9\,t)}{\pi} + \frac{4}{1089}\,\frac{\sin(33\,t)}{\pi} + \frac{4}{2401}\,\frac{\sin(49\,t)}{\pi} + \frac{4}{441}\,\frac{\sin(21\,t)}{\pi} - \frac{4}{121}\,\frac{\sin(11\,t)}{\pi} - \frac{4}{529}\,\frac{\sin(23\,t)}{\pi} \qquad (2.5)$$

$$A\ \frac{4}{289}\ \frac{\sin(17\,t)}{q}\ I\ \frac{4}{9}\ \frac{\sin(3\,t)}{q}\ I\ \frac{4}{961}\ \frac{\sin(31\,t)}{q}\ I\ \frac{4}{1225}\ \frac{\sin(35\,t)}{q}$$

$$A\ \frac{4}{1369}\ \frac{\sin(37\,t)}{q}\ A\ \frac{4}{625}\ \frac{\sin(25\,t)}{q}\ I\ \frac{4}{729}\ \frac{\sin(27\,t)}{q}\ A\ \frac{4}{841}\ \frac{\sin(29\,t)}{q}$$

$$A\ \frac{4\sin(t)}{q}\ A\ \frac{4}{25}\ \frac{\sin(5\,t)}{q}\ I\ \frac{4}{49}\ \frac{\sin(7\,t)}{q}\ A\ \frac{4}{169}\ \frac{\sin(13\,t)}{q}$$

$$I\ \frac{4}{361}\ \frac{\sin(19\,t)}{q}$$

Here the pattern is alternating signs for all odd numbers in teh sines summation.

```
M  f2o := (t,N) -> (4/Pi)*sum((-1)^k*sin((2*k+1)*t)/(2*k+1)^2, k=
   0..N);
```

$$f2o := (t, N) - \frac{4\left(\sum_{k=0}^{N} \frac{(I\ 1)^k \sin((2\,k A\ 1)\,t)}{(2\,k A\ 1)^2}\right)}{q} \quad (2.6)$$

```
M  simplify(f2o(t,24)-sinsum(f2,50,t));
```

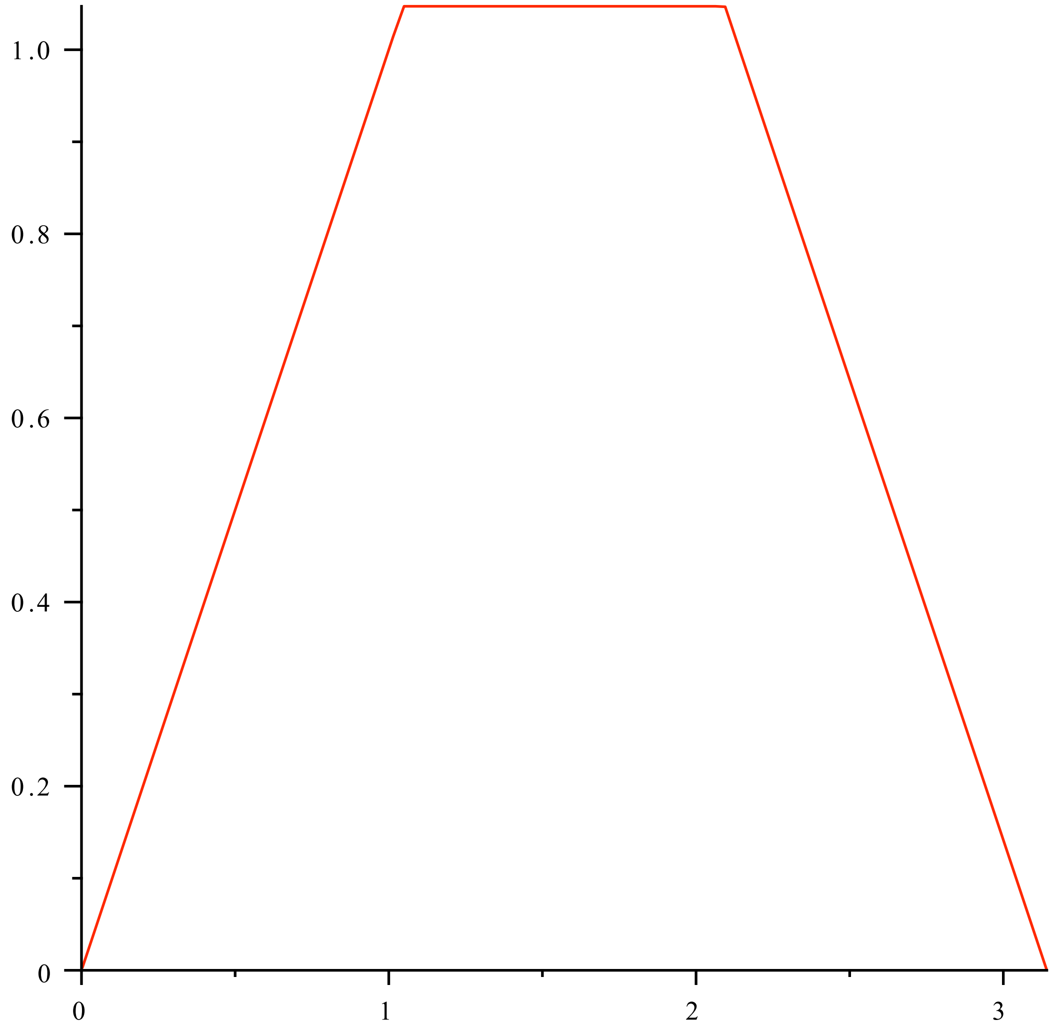
$$0 \quad (2.7)$$

Function from fig 9.2.5

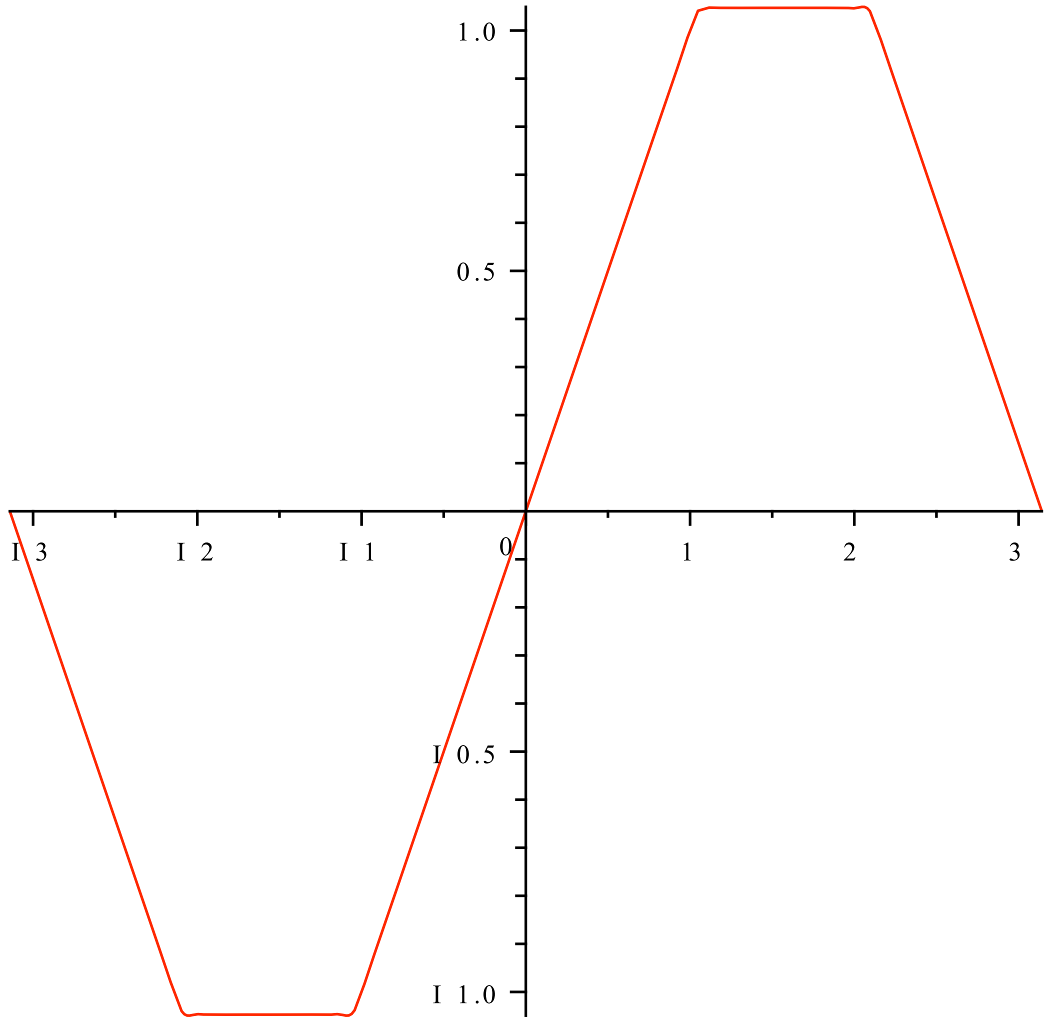
```
M  f3 := t-> unit(t,0,Pi/3)*t + unit(t,Pi/3,2*Pi/3)*(Pi/3) +
   (Pi-t)*unit(t,2*Pi/3,Pi);
```

$$f3 := t- \ \mathrm{unit}\left(t, 0, \frac{1}{3}\,q\right) t A\ \frac{1}{3}\ \mathrm{unit}\left(t, \frac{1}{3}\,q, \frac{2}{3}\,q\right) q A\ (q I\ \ t)\ \mathrm{unit}\left(t, \frac{2}{3}\,q, q\right) \quad (3.1)$$

```
M  plot(f3(t),t=0..Pi);
```




```
> plot(sinsum(f3,48,t),t=-Pi..Pi); sinsum(f3,48,t);
```



$$\frac{2\sqrt{3}\,\sin(t)}{\pi} - \frac{2}{25}\,\frac{\sqrt{3}\,\sin(5\,t)}{\pi} + \frac{2}{49}\,\frac{\sqrt{3}\,\sin(7\,t)}{\pi} - \frac{2}{121}\,\frac{\sqrt{3}\,\sin(11\,t)}{\pi} + \frac{2}{169}\,\frac{\sqrt{3}\,\sin(13\,t)}{\pi} - \frac{2}{289}\,\frac{\sqrt{3}\,\sin(17\,t)}{\pi} + \frac{2}{361}\,\frac{\sqrt{3}\,\sin(19\,t)}{\pi} - \frac{2}{529}\,\frac{\sqrt{3}\,\sin(23\,t)}{\pi} + \frac{2}{625}\,\frac{\sqrt{3}\,\sin(25\,t)}{\pi} - \frac{2}{841}\,\frac{\sqrt{3}\,\sin(29\,t)}{\pi} + \frac{2}{961}\,\frac{\sqrt{3}\,\sin(31\,t)}{\pi} - \frac{2}{1225}\,\frac{\sqrt{3}\,\sin(35\,t)}{\pi} + \frac{2}{1369}\,\frac{\sqrt{3}\,\sin(37\,t)}{\pi} - \frac{2}{1681}\,\frac{\sqrt{3}\,\sin(41\,t)}{\pi} + \frac{2}{1849}\,\frac{\sqrt{3}\,\sin(43\,t)}{\pi} - \frac{2}{2209}\,\frac{\sqrt{3}\,\sin(47\,t)}{\pi} \tag{3.2}$$

Here the pattern repeats every 6 terms

```
> f3o := (t,N) -> (2*sqrt(3)/Pi)*sum(sin((6*k+1)*t)/(6*k+1)^2 -
  sin((6*k+5)*t)/(6*k+5)^2, k=0..N);
```

(3.3)

$$f3o := (t, N) - \frac{2\sqrt{3}\left(\overset{N}{\underset{k=0}{<}}\left(\frac{\sin((6\,k A\ \ 1)\,t)}{(6\,k A\ \ 1)^2}\ l\ \ \frac{\sin((6\,k A\ \ 5)\,t)}{(6\,k A\ \ 5)^2}\right)\right)}{q} \tag{3.3}$$

Checking:

```
M  simplify(f3o(t,7)-sinsum(f3,48,t));
```

$$0 \tag{3.4}$$

```
M
```