Heated rod with ice bath (Example 9.5.2)

```
> L := 50; u0:=100; k:=0.15;
```

$$L := 50$$

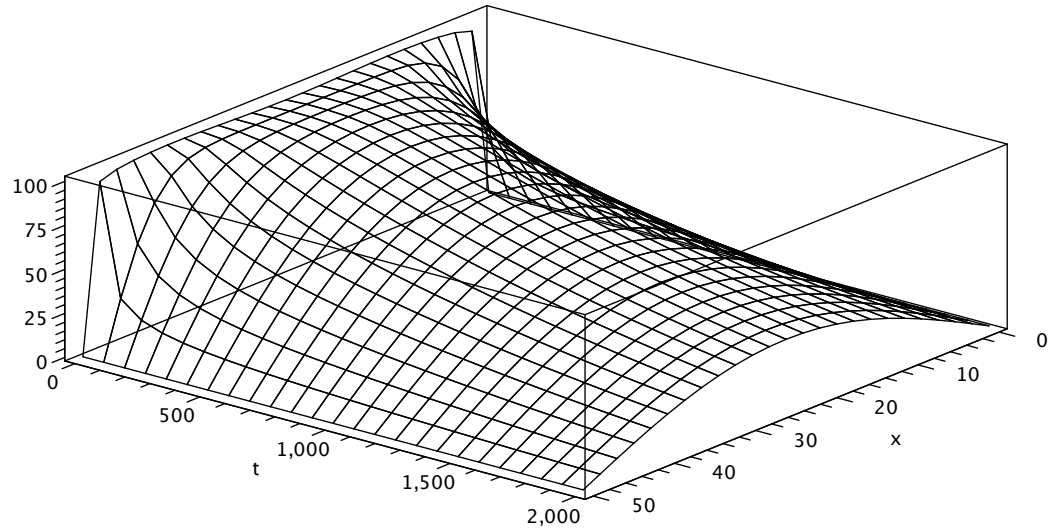
$$u0 := 100$$

$$k := 0.15 \quad \textbf{(1)}$$

```
> u := (x,t,N) -> (4*u0/Pi)* sum((1/(2*j-1)) * exp( -((2*j-1)*
  Pi/L)^2*k*t ) * sin((2*j-1)*Pi*x/L),j=1..N);
```

$$u := (x, t, N) \rightarrow \frac{4\,u0\left(\sum_{j=1}^{N} \frac{e^{-\frac{(2j-1)^2\pi^2 kt}{L^2}} \sin\left(\frac{(2j-1)\,\pi x}{L}\right)}{2j-1}\right)}{\pi} \quad \textbf{(2)}$$

```
> plot3d(u(x,t,50),x=0..L,t=0..2000,color=black,style=wireframe,
  axes=boxed);
```

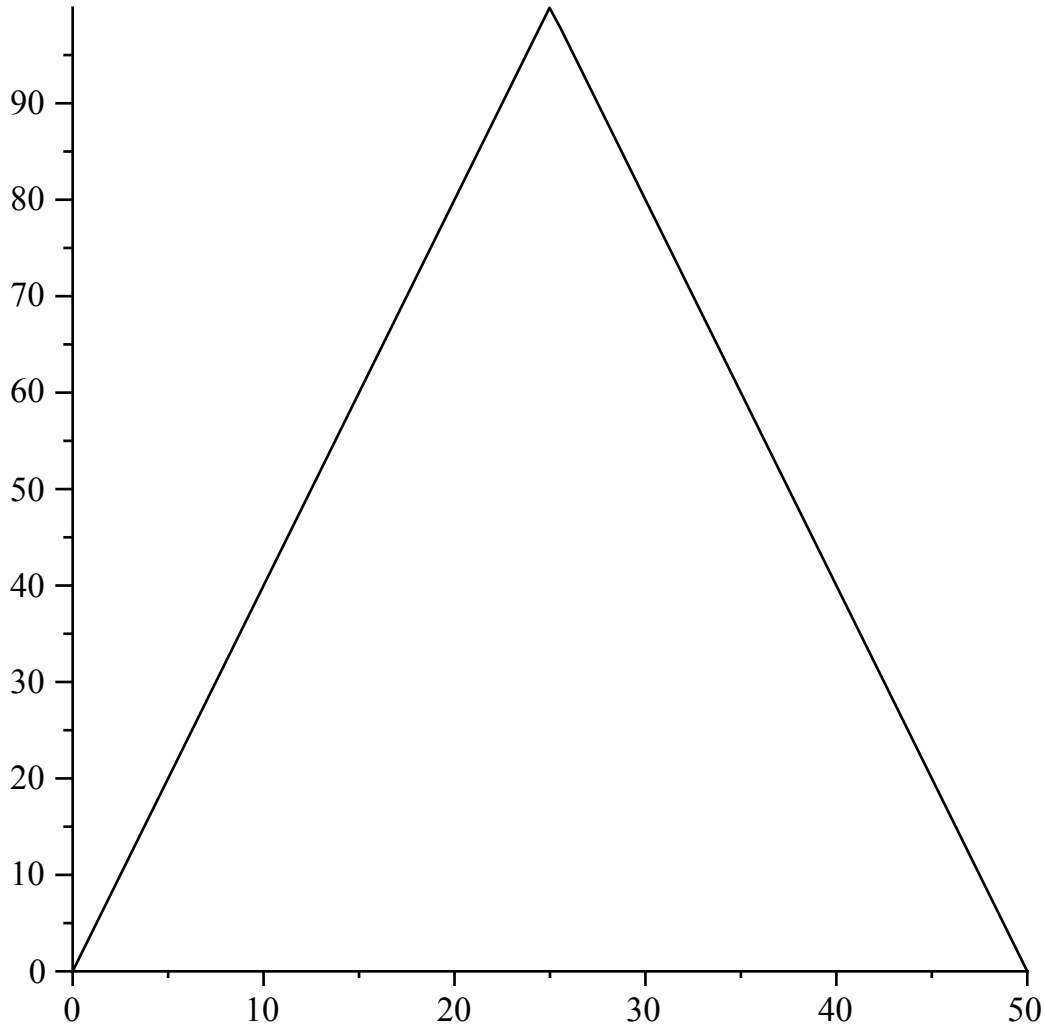


Heated rod with insulated ends (Example 9.5.3)

```
> f := x -> piecewise(0<x and x<=25, 4*x, 25<x and x<=50, 200-4*x)
  ;
```

$$f := x \rightarrow piecewise(0 < x \text{ and } x \le 25, 4x, 25 < x \text{ and } x \le 50, 200 - 4x) \quad \textbf{(3)}$$

```
> plot(f(x),x=0..50, color=black);
```



Fourier series of f(t)

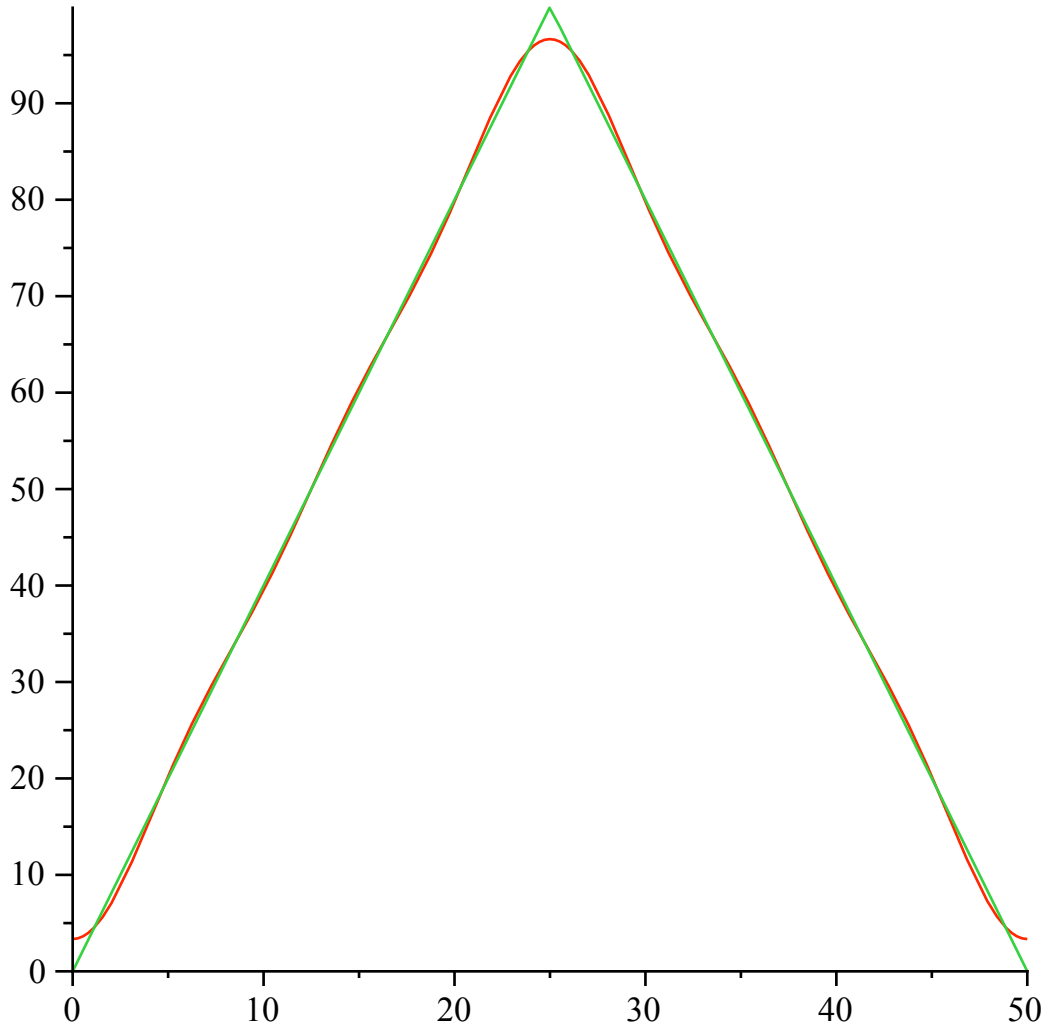
```
> a:= n-> (2/L) * int(f(x)*cos(n*Pi*x/L),x=0..L);
```

$$a := n \rightarrow \frac{2\left(\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)\mathrm{d}x\right)}{L} \tag{4}$$

```
> f2 := (x,N) -> a(0)/2 + sum(a(n)*cos(n*Pi*x/L),n=1..N);
```

$$f2 := (x, N) \rightarrow \frac{1}{2}a(0) + \sum_{n=1}^{N} a(n)\cos\left(\frac{n\pi x}{L}\right) \tag{5}$$

```
> plot({f2(x,10),f(x)},x=0..50);
```




```
> u2 := (x,t,N) -> a(0)/2 + sum(a(n) * exp(-(n*Pi/L)^2*k*t) * cos
  (n*Pi*x/L),n=1..N);
```

$$u2 := (x, t, N) \rightarrow \frac{1}{2} a(0) + \sum_{n=1}^{N} a(n)\, e^{-\frac{n^2 \pi^2 k t}{L^2}} \cos\left(\frac{n \pi x}{L} \right) \qquad (6)$$

```
> plot3d(u2(x,t,50),x=0..L,t=0..2000,color=black,style=wireframe,
  axes=boxed);
```