

Math 2280-2

Example 5.6.2 (three tanks cascade with salty water coming in)

```
> with(LinearAlgebra):
```

```
> A := Matrix(3,3,[-1/2,0,0, 1/2, -1/4, 0, 0, 1/4, -1/5]);
```

$$A := \begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & \frac{1}{4} & -\frac{1}{5} \end{bmatrix} \quad (1)$$

```
> xp := t -> Vector([40,80,100]);
```

$$xp := t \mapsto \text{Vector}([40, 80, 100]) \quad (2)$$

```
> Phi := t -> MatrixExponential(A*t);
```

$$\Phi := t \mapsto \text{LinearAlgebra}.\text{MatrixExponential}(A t) \quad (3)$$

```
> Phi(t);
```

$$\begin{bmatrix} e^{-\frac{1}{2}t} & 0 & 0 \\ 2e^{-\frac{1}{4}t} - 2e^{-\frac{1}{2}t} & e^{-\frac{1}{4}t} & 0 \\ \frac{25}{3}e^{-\frac{1}{5}t} - 10e^{-\frac{1}{4}t} + \frac{5}{3}e^{-\frac{1}{2}t} & -5e^{-\frac{1}{4}t} + 5e^{-\frac{1}{5}t} & e^{-\frac{1}{5}t} \end{bmatrix} \quad (4)$$

```
> c := -MatrixInverse(Phi(0)) . xp(0);
```

$$c := \begin{bmatrix} -40 \\ -80 \\ -100 \end{bmatrix} \quad (5)$$

```
> x := t -> Phi(t) . c + xp(t);
```

$$x := t \mapsto \Phi(t).c + xp(t) \quad (6)$$

```
> x(t);
```

$$\begin{bmatrix} -40e^{-\frac{1}{2}t} + 40 \\ -160e^{-\frac{1}{4}t} + 80e^{-\frac{1}{2}t} + 80 \\ -\frac{2500}{3}e^{-\frac{1}{5}t} + 800e^{-\frac{1}{4}t} - \frac{200}{3}e^{-\frac{1}{2}t} + 100 \end{bmatrix} \quad (7)$$

```
> plot(x(t), t=0..60, color=black);
```

