Math 2280-2
§5.4 Multiple Eigenvalue Solutions

Example 5.4.6

```
> with(LinearAlgebra):
  A:=Matrix(4,4,[0,0,1,0,  0,0,0,1,  -2,2,-3,1,  2,-2,1,-3]);
```

$$A := \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 2 & -3 & 1 \\ 2 & -2 & 1 & -3 \end{bmatrix} \quad (1)$$

The function of the package LinearAlgebra that computes eigenvectors (note capiltalization in Maple)

```
> Eigenvectors(A);
```

$$\begin{bmatrix} 0 \\ -2 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 & 0 & -\frac{1}{2} & 0 \\ 1 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (2)$$

The interpretation of the result we get is that 0 is a simple eigenvalue and -2 is an eigenvalue of algebraic multiplicity 3. Since we could only find 2 eigenvectors associated to lamba=-2, this means the matrix is defective.

The following shows that we only have two eigenvectors associated with lambda=-2

```
> Id:=Matrix(4,4,shape=identity);
> NullSpace(A+2*Id);
```

$$\left\{ \begin{bmatrix} 0 \\ -\frac{1}{2} \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -\frac{1}{2} \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\} \quad (3)$$

The algebraic multiplicity (multiplicity in the characteristic polynomial) of lambda=-2 is 3. Thus the generalized eigenspace is:

```
> NullSpace((A+2*Id)^3);
```

$$\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{1}{2} \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -\frac{1}{2} \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\} \quad (4)$$

However since the 'defect' or the number of 'missing' eigenvectors is one, we get exactly the same subspace if we look at

```
> NullSpace((A+2*Id)^2);
```

$$\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{1}{2} \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -\frac{1}{2} \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\} \tag{5}$$

This is because the defect tells you the size of the longest chain you can have. In this case chains no longer than 2 are already captured by NullSpace((A+2*Id)^2).

We take a vector in the generalized eigenspace for lambda=-2

```
> v:=Vector([0,0,1,-1]);
```

$$v := \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \tag{6}$$

We see we obtain a linear combination of the eigenvectors.

```
> u:=(A+2*Id).v;
```

$$u := \begin{bmatrix} 1 \\ -1 \\ -2 \\ 2 \end{bmatrix} \tag{7}$$

Thus (A+2*Id)*u = 0, and we have a chain since (A+2*Id)*v = u.