

Doomsday/Extinction example 01/27/2008

Assume a population is driven by DE

$$\begin{cases} \frac{dx}{dt} = x(x-1) \\ x(0) = 2 \end{cases}$$

Find time of doomsday (when population $\rightarrow \infty$ or "blows up")

This is a separable equation so:

$$\int \frac{-dx}{x(x-1)} = -\int dt + C$$

$$\int \left(\frac{1}{x} - \frac{1}{x-1} \right) dx = \frac{\ln|x|}{\ln|x-1|}$$

Assuming $x > 1$ we get:

$$\frac{x}{x-1} = \underbrace{A}_{\text{some } > 0 \text{ constant}} e^{-t}$$

From $x(0) = 2$ we get:

$$\frac{x(0)}{x(0)-1} = A$$
$$\frac{2}{2-1}$$

$$\text{Thus: } \frac{x}{x-1} = 2e^{-t} \Rightarrow x = \frac{2e^{-t}}{1-2e^{-t}} = \frac{2}{e^t - 2}$$

thus $x(t) \rightarrow \infty$ when $t \rightarrow \ln 2$.