

## Math 2280-2 Free damped motions

The kind of motion depends on the sign of  $p^2 - w_0^2$ . See the textbook pp190-192.

### Underdamped free motion

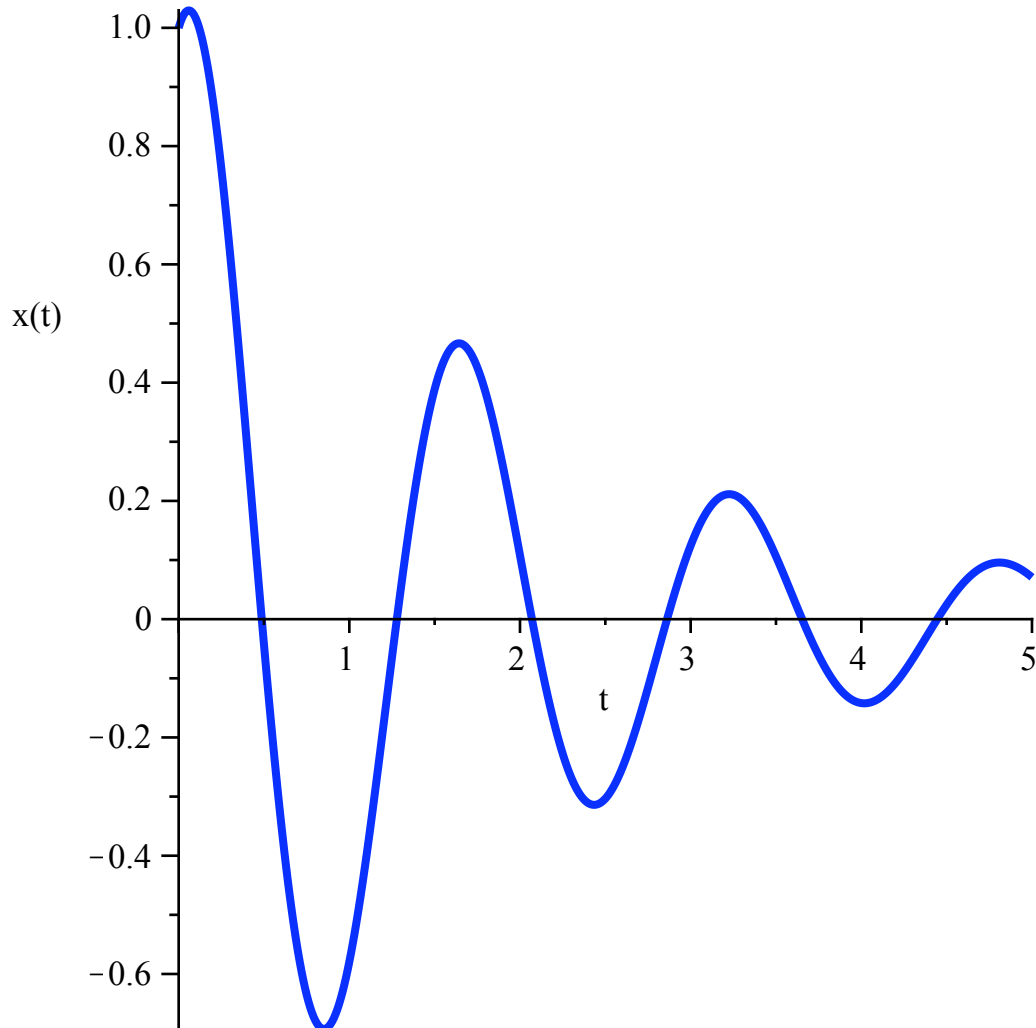
```
> with(DEtools):  
> p:=0.5; w0:=4;  
deqtn:= diff(x(t),t,t) + 2*p*diff(x(t),t) + w0^2*x(t) = 0;  
# note the syntax for double derivatives  
      p := 0.5  
      w0 := 4
```

$$\text{deqtn} := \frac{d^2}{dt^2} x(t) + 1.0 \left( \frac{d}{dt} x(t) \right) + 16 x(t) = 0 \quad (1)$$

```
> dsolve(deqtn);
```

$$x(t) = \_C1 e^{-\frac{1}{2}t} \sin\left(\frac{3}{2}\sqrt{7}t\right) + \_C2 e^{-\frac{1}{2}t} \cos\left(\frac{3}{2}\sqrt{7}t\right) \quad (2)$$

```
> DEplot(deqtn,x(t),t=0..5,[[x(0)=1,D(x)(0)=1]],stepsize=1/100,  
linecolor=[blue]);  
# note the syntax of the initial condition
```



## Critically damped free motion

This case

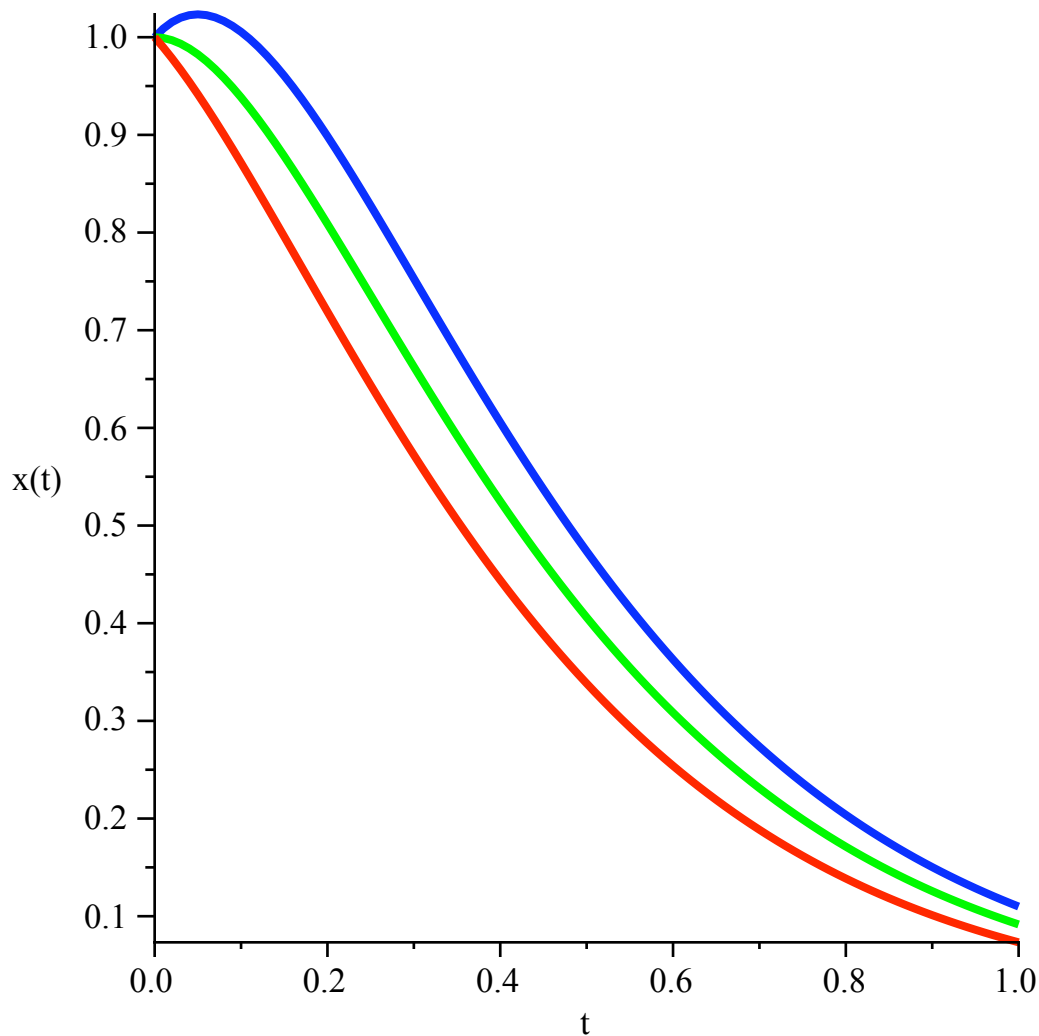
```
> p:=4; w0:=4;  
deqtn:= diff(x(t),t,t) + 2*p*diff(x(t),t) + w0^2*x(t) = 0;  
p:=4  
w0:=4
```

$$deqtn := \frac{d^2}{dt^2} x(t) + 8 \left( \frac{d}{dt} x(t) \right) + 16 x(t) = 0 \quad (3)$$

```
> dsolve(deqtn);
```

$$x(t) = \_C1 e^{-4t} + \_C2 e^{-4t} t \quad (4)$$

```
> DEplot(deqtn,x(t),t=0..1,[[x(0)=1,D(x)(0)=1],[x(0)=1,D(x)(0)=0],  
[x(0)=1,D(x)(0)=-1]],stepsize=1/100,linecolor=[blue,green,red]);
```



## Damped free motion

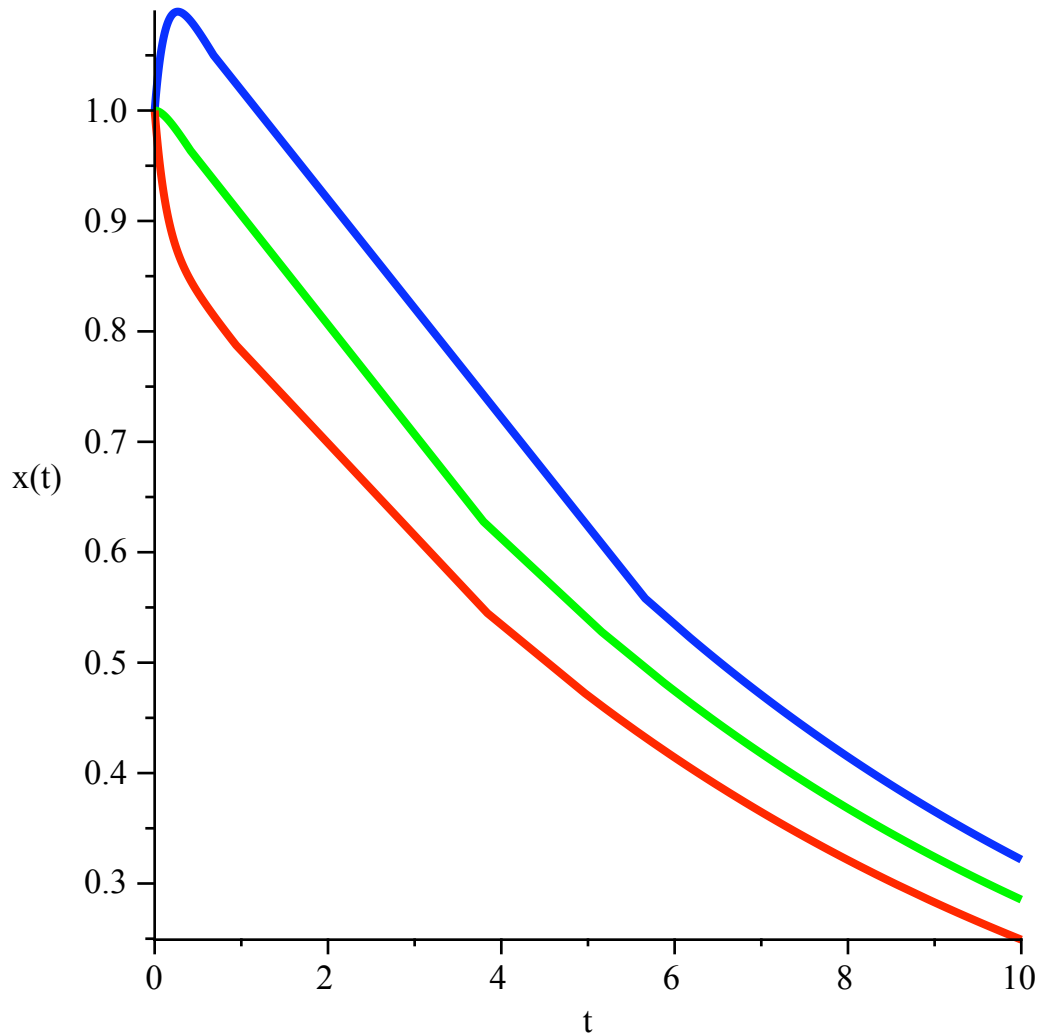
```
> p:=4; w0:=1;  
deqtn:= diff(x(t),t,t) + 2*p*diff(x(t),t) + w0^2*x(t) = 0;  
p:=4  
w0:=1
```

$$\text{deqtn} := \frac{d^2}{dt^2} x(t) + 8 \left( \frac{d}{dt} x(t) \right) + x(t) = 0 \quad (5)$$

```
> dsolve(deqtn);
```

$$x(t) = \_C1 e^{(-4 + \sqrt{15})t} + \_C2 e^{-(4 + \sqrt{15})t} \quad (6)$$

```
> DEplot(deqtn, x(t), t=0..10, [[x(0)=1, D(x)(0)=1], [x(0)=1, D(x)(0)=0],  
[x(0)=1, D(x)(0)=-1]], stepsize=1/200, linecolor=[blue, green, red])  
;
```



```
>
```