MATH 205: Homework 6

Due Friday May 11

There are problems from Boller and Sally and also from "Multivariable Mathematics" by T. Shifrin. I recommend at least thinking about all the exercises, even if they are not assigned, as you read through the textbook. I also recommend trying to prove all the Theorems which are left unproven in the book. **There may be more problems assigned depending on our progress in lecture**

Problem 1. Boller and Sally, Section 6.3: 5,6

Problem 2. Shifrin, Section 8.2 Exercises: 2, 3, 4, 6abc, 7abce, 8abcd, 10

Problem 3. Let C be an oriented curve in $\mathbb{R}^2$ and let n the unit outward normal (i.e. $\{n, T\}$ is a right handed basis for $\mathbb{R}^2$, where T is the tangent to C). Let $F = (F_1, F_2)$ be a vector field in $\mathbb{R}^2$ and $\omega = F_1 dx + F_2 dy$ be the corresponding 1-form. Show that

$$\int_C F \cdot n \; ds = \int_C F_1 dy - F_2 dx.$$

Here $\int_C f \; ds$ is the arc-length integral defined by $\int_a^b f(\gamma(t))|\gamma'(t)|dt$ if γ parametrizes C. The quantity on the left is called the *flux* of the vector field F through C. Conclude that if $C = \partial\Omega$ then,

$$\int_C F \cdot n ds = \int_\Omega \nabla \cdot F = \int_\Omega \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y}.$$

Problem 4. An ant finds himself in the xy-plane in the presence of the force field $F = (y^3 + x^2 y, 2x^2 - 6xy)$. Around what simple closed curve should he travel counter-clockwise inorder to maximize the work done on him by F? (Hint: use Green's theorem)