MATH 205: Homework 3

Due Wednesday Oct 18

Problems are from Boller and Sally. I recommend at least thinking about all the exercises, even if they are not assigned, as you read through the textbook. I also recommend trying to prove all the Theorems which are left unproven in the book.

Problem 1. Exercises 5.10.3, 5.10.6, 5.10.7, 5.10.8

Problem 2. Suppose $E \subset \mathbb{R}^n$ is a bounded set such that, for all $\varepsilon > 0$ there exists an open set $U \supset E$, with ∂U having measure zero, such that $|U| \leq \varepsilon$. Show that E has measure zero.

Problem 3. Let R be a closed bounded rectangle. and $f : R \to \mathbb{R}$ bounded.

(a) Suppose $f: R \to \mathbb{R}$ bounded. Show that if $E = \{x \in R : f \neq 0\}$ has measure zero then f is Riemann integrable on R and,

$$\int_R f \, dx = 0$$

(b) Suppose that $f, g: R \to \mathbb{R}$ are bounded and integrable. Show that if $D = \{x \in R: f(x) \neq g(x)\}$ has measure zero then,

$$\int_R f \, dx = \int_R g \, dx.$$

(c) Show that if $f: R \to \mathbb{R}$ is Riemann integrable and $\Omega \subset R$ is an open set with $\partial \Omega$ having measure zero then,

$$\int_{\Omega} f \, dx = \int_{\overline{\Omega}} f \, dx = \int_{\Omega^0} f \, dx$$

Problem 4. Cylindrical coordinates (r, θ, z) for \mathbb{R}^3 are defined by $(x, y, z) = (r \cos \theta, r \sin \theta, z)$. Spherical coordinates were defined in the previous problem 5.10.6. Compute the following integrals by choosing a convenient coordinate system.

- (a) $\int_D z^2$ where D is the hemisphere $D = \{x^2 + y^2 + z^2 \le 1\} \cap \{z \ge 0\}.$ (b) $\int_{-3}^3 \int_0^{\sqrt{9-x^2}} \int_{\sqrt{x^2+y^2}}^3 xy \ dz dy dx$ (what region is this integral over?)

Problem 5. Exercises 6.1.10, 6.1.11, 6.1.13