MATH 203: Homework 2

Due Wednesday Oct 12

Problems are from Rudin 3rd edition.

Problem 1. 2, 3, 4, 6, 7, 8, 9

Problem 2. Establish an explicit bijection between the two sets:

(*i*) The natural numbers $\mathbb{N} = \{0, 1, 2, \dots\}$ and the positive natural numbers $\mathbb{N}_+ = \{1, 2, \dots\}$.
(*ii*) The open interval $(0, 1)$ and $\mathbb{R}$ (a familiar function from calculus may help).
(*iii*) The open interval $(0, 1)$ and the closed interval $[0, 1]$.

Problem 3. Metric spaces.

(*i*) Consider the set $\mathbb{R}^2$ with the taxi-cab (or ℓ^1) metric,

$$d_1(x, y) = |x_1 - y_1| + |x_2 - y_2|.$$

What is the open unit ball $B(0, 1)$ in the metric d_1? Draw a picture.

(*ii*) Let X be an arbitrary set, and define the discrete metric d_0 on X by $d_0(x, y) = 0$ if $x = y$ and $d_0(x, y) = 1$ if $x \neq y$. Let $x \in X$, identify the open ball $B(x, 1)$, the closed ball $\{y \in X : d_0(x, y) \leq 1\}$, and the closure of the open unit ball $\overline{B(x, 1)}$.